Limits, Continuity and Differentiability

Question1

$$\lim_{x \to \infty} [x - \log(\cosh x)] =$$

AP EAPCET 2025 - 26th May Morning Shift

Options:

A.

2

B.

0

C.

$\log \frac{1}{2}$

D.

$\log 2$

Answer: D

Solution:

$$\lim_{x \to \infty} [x - \log(\cosh x)]$$

$$\because \quad \cosh x = \frac{e^x + e^{-x}}{2}$$

$\because x \to \infty, e^{-x}$ become negligible

$$\therefore \cosh x \simeq \frac{e^x}{2}$$

$$\Rightarrow \log(\cosh x) = \log\left(\frac{e^x}{2}\right) = x - \log 2$$

$$= \lim_{x \to \infty}[x - x + \log 2] = \log 2$$

Question2

$$\lim_{x \to \infty}\left(\sqrt[3]{x^3 + 4x^2} - \sqrt{x^2 - 3x}\right) =$$

AP EAPCET 2025 - 26th May Morning Shift

Options:

A.

$\frac{17}{6}$

B.

$\frac{25}{6}$

C.

$-\frac{1}{6}$

D.

$\frac{37}{6}$

Answer: A

Solution:

$$\lim_{x \to \infty}\left(\sqrt[3]{x^3 + 4x^2} - \sqrt{x^2 - 3x}\right)$$

$$= \lim_{x \to \infty}\left(x\left(\sqrt[3]{1 + \frac{4}{x}} - \sqrt{1 - \frac{3}{x}}\right)\right)$$

Apply binomial expansion for radicals

$$= \lim_{x\to\infty}\left(x\left(\left(1+\frac{1}{3}\cdot\frac{4}{x}+0\left(\frac{1}{x^2}\right)\right)-\left(1-\frac{1}{2}\cdot\frac{3}{x}+0\left(\frac{1}{x^2}\right)\right)\right)\right)$$

$$= \lim_{x\to\infty} x\left(1+\frac{4}{3x}-1+\frac{3}{2x}\right)$$

$$= \lim_{x\to\infty} x\cdot\frac{1}{x}\left(\frac{4}{3}+\frac{3}{2}\right)$$

$$= \frac{4}{3}+\frac{3}{2}=\frac{8+9}{6}=17/6$$

Question3

If a real valued function $f(x)=\begin{cases} e^{\frac{\sin a(x-[x])}{x-[x]}} & , \text{ if } x<1 \\ b+1 & , \text{ if } x=1 \\ \frac{|x^2+x-2|}{x-1} & , \text{ if } x>1 \end{cases}$ **is continuous at** $x=1$**, then** $b\sin a=$ **(**$[x]$ **denotes the greatest integer function)**

AP EAPCET 2025 - 26th May Morning Shift

Options:

A.

6

B.

4

C.

$\log_e 9$

D.

$\log_6 2$

$\lim_{x\to 1^-}$

Answer: C

Solution:

$\because f(x)$ is continuous at $x = 1$

$\therefore \lim_{x \to 1^-} f(x) = f(1) = \lim_{x \to 1^+} f(x)$

So, $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \left[e^{\frac{\sin a(x - [x])}{x - [x]}} \right]$

$= \lim_{x \to 1^-} \left(e^{\frac{\sin a(x-0)}{x-0}} \right) = e^{\sin a}$

and $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \frac{|x^2 + x - 2|}{x - 1}$

$= \lim_{x \to 1^+} \frac{(x-1)(x+2)}{(x-1)} = \lim_{x \to 1^+} (x + 2) = 3$

and $\quad f(1) = b + 1 = 3$

$\Rightarrow b = 2$ and $e^{\sin a} = 3 \Rightarrow \sin a = \ln 3$

Hence, $b \sin a = 2 \ln 3 = \ln 9$

Question4

$$\lim_{x \to 0} \frac{x + 2\sin x + 3\tan x - \tan^3 x}{\sqrt{x^2 + 2\sin x + \tan x + 3} - \sqrt{\sin^2 x - 2\tan x - x + 3}}$$

AP EAPCET 2025 - 26th May Evening Shift

Options:

A.

$2\sqrt{3}$

B.

10

C.

25

D.

$\sqrt{17}$

Answer: A

Solution:

$$\lim_{x \to 0} \frac{x+2\sin x+3\tan x-\tan^3 x}{\sqrt{x^2+2\sin x+\tan x+3-\sqrt{\sin^2 x-2\tan x-x+3}}}$$

After rationalisation

$$\left(x+2\sin x+3\tan x-\tan^3 x\right)$$

$$\lim_{x \to 0} \frac{\left(\sqrt{x^2+2\sin x+\tan x+3}+\sqrt{\sin^2 x-2\tan x-x+3}\right.}{x^2+2\sin x+\tan x+3-\sin^2 x+2\tan x+x-3}$$

$$x\left(1+\frac{2\sin x}{x}+\frac{3\tan x}{x}-\frac{3\tan^3 x}{x}\right)$$

$$\lim_{x \to 0} \frac{\left(\sqrt{x^2+2\sin x+\tan x+3}+\sqrt{\sin^2 x-2\tan x-x+3}\right)}{x\left(1+x+\frac{2\sin x}{x}+\frac{\tan x}{x}-\frac{\sin^2 x}{x}+\frac{2\tan x}{x}\right)}$$

$$\Rightarrow \frac{(1+2+3)(\sqrt{3}+\sqrt{3})}{(1+2+1+2)} = \frac{6\times 2\sqrt{3}}{6} = 2\sqrt{3}$$

Question5

$$\lim_{x \to \infty} \frac{(3-x)^{25}(6+x)^{35}}{(12+x)^{38}(9-x)^{22}} =$$

AP EAPCET 2025 - 26th May Evening Shift

Options:

A.

3^{60}

B.

-1

C.

1

D.

0

Answer: B

Solution:

Given,

$$\lim_{x \to \infty} \frac{(3-x)^{25}(6+x)^{35}}{(12+x)^{38}(9-x)^{22}}$$

$$= \lim_{x \to \infty} \frac{x^{25} \cdot x^{35}\left(\frac{3}{x}-1\right)^{25}\left(\frac{6}{x}+1\right)^{35}}{x^{38} \cdot x^{22}\left(\frac{12}{x}+1\right)^{38}\left(\frac{9}{x}-1\right)^{22}}$$

$$= \lim_{x \to \infty} \frac{\left(\frac{3}{x}-1\right)^{25}\left(\frac{6}{x}+1\right)^{35}}{\left(\frac{12}{x}+1\right)^{38}\left(\frac{9}{x}-1\right)^{22}}$$

$$= \frac{(-1)(1)}{(1)(1)} = -1 \left\{ \because \begin{array}{c} x \to \infty \\ \frac{3}{x} \to 0 \\ \frac{6}{x} \to 0 \\ \frac{12}{x} \to 0 \\ \frac{9}{x} \to 0 \end{array} \right\}$$

Question6

If a real valued function

$$f(x) = \begin{cases} \log(1+[x]), & x \geq 0 \\ \sin^{-1}[x], & -1 \leq x < 0 \\ k([x]+|x|), & x < -1 \end{cases}$$

is continuous at $x = -1$, then $k =$

AP EAPCET 2025 - 26th May Evening Shift

Options:

A.

$-\pi/2$

B.

$-\pi$

C.

π

D.

$\pi/2$

Answer: D

Solution:

We have,

$$f(x)=\begin{cases}\log(1+[x]) & x\geq 0\\ \sin^{-1}[x] & -1\leq x<0\\ k([x]+|x|] & x<-1\end{cases}$$

$\because f(x)$ is continuous at $x=-1$

$$\begin{aligned}&\therefore f(-1)=f\left(-1^{-}\right)=f\left(-1^{+}\right)\\&\sin^{-1}(-1)=k\left(\left[-1^{-}\right]+\left|-1^{-}\right|\right)=\sin^{-1}\left[-1^{+}\right]\\&\Rightarrow\quad \frac{-\pi}{2}=k(-2+1)=\frac{-\pi}{2}\\&\Rightarrow\quad -k=-\pi/2\\&\therefore\quad k=\frac{\pi}{2}\end{aligned}$$

Question7

$$\lim_{n\to\infty}\frac{\pi}{2n}\left[\sin\frac{\pi}{2n}+\sin\frac{2\pi}{2n}+\sin\frac{3\pi}{2n}+\ldots+\sin\frac{\pi}{2}\right]=$$

AP EAPCET 2025 - 26th May Evening Shift

Options:

A.

1

B.

0

C.

4

D.

3

Answer: A

Solution:

$$\lim_{n\to\infty} \frac{\pi}{2n}\left[\sin\frac{\pi}{2n}+\sin\frac{2\pi}{2n}+\sin\frac{3\pi}{2n}+\ldots+\sin\frac{\pi}{2}\right]$$

$$=\lim_{n\to\infty}\frac{\pi}{2n}\left(\frac{\sin\frac{n}{2}\left(\frac{\pi}{2n}\right)}{\sin\frac{1}{2}\left(\frac{\pi}{2n}\right)}\cdot\sin\left(\frac{\frac{\pi}{2n}+\frac{\pi}{2}}{2}\right)\right)$$

$$=\lim_{n\to\infty}\frac{\pi}{2n}\cdot\frac{\left(\sin\frac{\pi}{4}\right)}{\sin\frac{\pi}{4n}}\cdot\sin\left(\frac{\pi}{4n}+\frac{\pi}{4}\right)$$

$$=\lim_{n\to\infty}\frac{\pi}{2n}\frac{(1/\sqrt{2})}{\left(\frac{\sin\frac{\pi}{4n}}{\frac{\pi}{4n}}\right)}\cdot\sin\left(\frac{\pi}{4n}+\frac{\pi}{4}\right)$$

$$=2\cdot\frac{1}{\sqrt{2}}\sin\pi/4=1$$

Question8

$[x]$ represents the greatest integer function. If $\lim\limits_{x\to0+}\frac{\cos[x]-\cos(kx-[x])}{x^2}=5$, then $k=$

AP EAPCET 2025 - 24th May Morning Shift

Options:

A.

$\sqrt{10}$

B.

$\sqrt{11}$

C.

3

D.

9

Answer: A

Solution:

Since, $x \to 0^+$, so, the greatest integer function $[x]$ equals 0 as $0 \leq x < 1$.

Now, $\lim\limits_{x \to 0+} \frac{\cos[x] - \cos(kx - [x])}{x^2} = 5$

$\Rightarrow \quad \lim\limits_{x \to 0^+} \frac{\cos(0) - \cos(kx - 0)}{x^2} = 5 \Rightarrow$

$$\lim_{x \to 0^+} \frac{1 - \cos(kx)}{x^2} = 5$$

$$\Rightarrow \quad \lim_{x \to 0^+} \frac{2\sin^2\left(\frac{kx}{2}\right)}{x^2} = 5$$

$$\Rightarrow \lim_{x \to 0^+} 2\frac{\sin^2\left(\frac{kx}{2}\right)}{\left(\frac{kx}{2}\right)^2} \cdot \frac{k^2}{4} = 5$$

$$\Rightarrow \quad 2 \cdot 1 \cdot \frac{k^2}{4} = 5 \quad \left[\because \lim_{\theta \to 0} \frac{\sin(\theta)}{\theta} = 1\right]$$

$$\Rightarrow \quad k^2 = \frac{20}{2} \Rightarrow k = \pm\sqrt{10}$$

So, $k = \sqrt{10}$

Question9

$\lim\limits_{x \to 0} \frac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2} =$

AP EAPCET 2025 - 24th May Morning Shift

Options:

A.

$-\frac{1}{2}$

B.

$\frac{1}{2}$

C.

$\frac{1}{4}$

D.

1

Answer: B

Solution:

$$
\begin{aligned}
&\lim_{x \to 0} \frac{x \tan 2x - 2x \tan x}{(1-\cos 2x)^2} \\
&= \lim_{x \to 0} \frac{x \cdot \frac{2\tan x}{1-\tan^2 x} - 2x \tan x}{\left(1-\left(1-2\sin^2 x\right)\right)^2} \\
&= \lim_{x \to 0} \frac{2x \tan x \left(\frac{1}{1-\tan^2 x} - 1\right)}{\left(2\sin^2 x\right)^2} \\
&= \lim_{x \to 0} \frac{\frac{2x \tan x\left(1-1+\tan^2 x\right)}{1-\tan^2 x}}{4\sin^4 x} = \lim_{x \to 0} \frac{2x \tan x \cdot \tan^2 x}{4\sin^4 x\left(1-\tan^2 x\right)} \\
&= \lim_{x \to 0} \frac{x \tan^3 x}{2\sin^4 x\left(1-\tan^2 x\right)} = \lim_{x \to 0} \frac{x \frac{\sin^3 x}{\cos^3 x}}{2\left(1-\tan^2 x\right) \cdot \cos^2 x \cdot \sin^2 x} \\
&= \lim_{x \to 0} \frac{1}{2\cos^3 x\left(1-\tan^2 x\right)} \cdot \lim_{x \to 0} \frac{x}{\sin x}
\end{aligned}
$$

$= \frac{1}{2\cos^3 0\left(1-\tan^2 0\right)} \cdot 1 \quad \left[\because \lim_{x \to 0} \frac{x}{\sin x} = 1\right]$

$= \frac{1}{2(1)(1)} = \frac{1}{2}$

Question10

If $f(x) = \begin{cases} \frac{\left(e^{ax}-1\right)\log(1+x)}{\sin^2 x}, & \text{if } x > 0 \\ 2, & \text{if } x = 0 \\ \frac{\cos 4x - \cos bx}{\tan^2 x}, & \text{if } x < 0 \end{cases}$ **is continuous at** $x = 0$**, then**

$\sqrt{b^2 - a^2} =$

Options:

A.

4

B.

5

C.

3

D.

7

Answer: A

Solution:

Given, function

$$f(x) = \begin{cases} \frac{(e^{ax}-1)\log(1+x)}{\sin^2 x} & , \quad x > 0 \\ \frac{\cos 4x - \cos bx}{\tan^2 x} & , \quad x = 0 \end{cases}$$

is contiuous at $x = 0$

Since, $f(x)$ is continuous at $x = 0$

So, $\lim\limits_{x \to 0-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$ and $f(0) = 2$

Now, $\lim\limits_{x \to 0+} \frac{(e^{ax}-1)\log(1+x)}{\sin^2 x}$

Applying L'hospital rule, twice, since it is an indeterminate form $\frac{0}{0}$.

So, $\lim\limits_{x \to 0+} \frac{ae^{ax}\log(1+x) + \frac{(e^{ax}-1)}{1+x}}{2\cos x \sin x}$

This limit is again $\frac{0}{0}$ form, so apply L'Hospital rule again.

$$\lim_{x \to 0+} \frac{a^2 e^{ax}\log(1+x) + \frac{ae^{ax}(1+x)-(e^{ax}-1)}{(1+x)^2} + \frac{ae^{ax}}{1+x}}{2\cos^2 x - 2\sin^2 x}$$

$= \frac{0+\frac{a(1)-(1-1)}{1^2}+\frac{a}{1}}{2(1)-0} = \frac{a+a}{2} = a$

Since, $\lim\limits_{x \to 0+} f(x) = f(0) \Rightarrow a = 2$

Now, $\lim\limits_{x \to 0-} \frac{\cos 4x - \cos bx}{\tan^2 x}$

$= \lim\limits_{x \to 0-} \frac{-2 \sin\left(\frac{4x+bx}{2}\right) \sin\left(\frac{4x-bx}{2}\right)}{\tan^2 x}$

$$= \lim_{x \to 0^-} \frac{-2 \sin\left(\frac{4x+bx}{2}\right) \sin\left(\frac{4x-bx}{2}\right) \cdot \cos^2 x}{\frac{\sin^2 x}{x^2} \cdot x^2}$$

$$\lim_{x \to 0^-} \frac{\frac{-2 \sin\left(\frac{4x+bx}{2}\right)}{\frac{4x+bx}{2}} \cdot \left(\frac{4x+bx}{2}\right) \cdot \frac{\sin\left(\frac{4x-bx}{2}\right)}{\left(\frac{4x-bx}{2}\right)} \left(\frac{4x-bx}{2}\right) \cdot \cos^2 x}{\frac{\sin^2 x}{x^2} \cdot x^2}$$

$$= \lim_{x \to 0^-} \frac{-2 \cdot 1 \cdot \left(\frac{4x+bx}{2}\right) \cdot 1 \cdot \left(\frac{4x-bx}{2}\right) \cdot \cos^2 x}{1 \cdot x^2} \quad \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1\right]$$

$$= \lim_{x \to 0^-} \frac{-2\left(16x^2 - b^2 x^2\right)}{4x^2} \cdot \cos^2 x$$

$$= \lim_{x \to 0^-} \frac{\left(b^2 - 16\right)}{2} \cdot \cos^2 x = \frac{b^2 - 16}{2} \cdot 1 \quad \left[\because \lim_{x \to 0^-} \cos^2 x = 1\right]$$

$= \frac{b^2-16}{2}$

Also, $\lim\limits_{x \to 0-} f(x) = f(0)$

$\Rightarrow \quad \frac{b^2-16}{2} = 2 \Rightarrow b^2 = 4 + 16 = 20$

Now, $\sqrt{b^2 - a^2} = \sqrt{20 - 2^2} = \sqrt{20 - 4} = \sqrt{16} = 4$

Question11

$$\lim_{x \to 0} \frac{x^2 \sin^2(3x) + \sin^4(6x)}{(1-\cos 3x)^2} =$$

AP EAPCET 2025 - 23rd May Evening Shift

Options:

A.

$\frac{580}{9}$

B.

$\frac{145}{3}$

C.

$\frac{580}{3}$

D.

$\frac{145}{9}$

Answer: A

Solution:

$$\lim_{x\to 0}\frac{x^2\sin^2(3x)+\sin^4(6x)}{(1-\cos 3x)^2}$$
$$=\lim_{x\to 0}\frac{x^2\sin^2(3x)+\sin^4(6x)}{\left(2\sin^2\left(\frac{3x}{2}\right)\right)^2}$$

For small-angles $\{\because x\to 0\}$

$\sin kx\to kx$

$$=\lim_{x\to 0}\frac{x^2(3x)^2+(6x)^4}{\left(2\left(\frac{3x}{2}\right)^2\right)^2}$$
$$=\lim_{x\to 0}\frac{x^4[9+1296]}{x^4\left[\frac{9}{2}\right]^2}=\lim_{x\to 0}\frac{1305}{81}\times 4$$
$$=\frac{435}{27}\times 4=\frac{580}{9}$$

Question12

If a real valued function

$$f(x)=\begin{cases}(1+\sin x)^{\cos x}, & -\pi/2<x<0\\ a, & x=0\\ \frac{e^{2/x}+e^{3/x}}{ae^{2/x}+be^{3/x}}, & 0<x<\pi/2\end{cases}$$

is continuous at $x=0$, then $ab=$

AP EAPCET 2025 - 23rd May Evening Shift

Options:

A.

e

B.

e^2

C.

1

D.

-1

Answer: C

Solution:

$$f(x)=\begin{cases}(1+\sin x)^{\cos x}, & -\frac{\pi}{2}<x<0\\ a, & x=0\\ \frac{e^{\frac{2}{x}}+e^{\frac{3}{x}}}{ae^{\frac{2}{x}}+be^{\frac{3}{x}}}, & 0<x<\frac{\pi}{2}\end{cases}$$

So, $\lim\limits_{x\to0^-} f(x)=\lim_{x\to0^-}(1+\sin x)^{\cos x}$

Let $y=(1+\sin x)^{\cos x}$

$$\Rightarrow \ln y=\cos x\ln(1+\sin x)$$
$$=\frac{\ln(1+\sin x)}{\sin x}=1$$

$\Rightarrow \quad y=e$

$\therefore \lim\limits_{x\to0^-}(1+\sin x)^{\cos x}=e=a=f(0)$

and $\lim\limits_{x\to0^+} f(x)=\lim\limits_{x\to0^+}\frac{e^{\frac{2}{x}}+e^{\frac{3}{x}}}{ae^{\frac{2}{x}}+be^{\frac{3}{2}}}$

$=\lim\limits_{x\to0^+}\frac{e^{\frac{3}{x}}\left(e^{-\frac{1}{x}}+1\right)}{e^{\frac{3}{x}}\left(e\cdot e^{\frac{-1}{x}}+b\right)}=\frac{1}{b}$

So, $e=\frac{1}{b}\Rightarrow b=\frac{1}{e}$

Hence, $ab=e\cdot\frac{1}{e}=1$

Question13

$$\lim_{x \to 0} \frac{(\operatorname{cosec} x - \cot x)(e^x - e^{-x})}{\sqrt{3} - \sqrt{2 + \cos x}} =$$

AP EAPCET 2025 - 23rd May Evening Shift

Options:

A.

$3\sqrt{2}$

B.

$2\sqrt{3}$

C.

$3\sqrt{3}$

D.

$4\sqrt{3}$

Answer: D

Solution:

$\lim_{x \to 0} \frac{(\operatorname{cosec} x - \cot x)(e^x - e^{-x})}{\sqrt{3} - \sqrt{2 + \cos x}}$

Convert, $\operatorname{cosec} x$ and $\cot x$ into $\sin x$ and $e^x - e^{-x} = 2 \sin hx$

$$= \lim_{x \to 0} \frac{\left(\frac{2 \sin^2 \frac{x}{2}}{2 \sin \frac{x}{2} \cdot \cos \frac{x}{2}} \cdot 2 \sin hx\right)}{\sqrt{3} - \sqrt{2 + \cos x}}$$

$$= \lim_{x \to 0} \frac{2 \tan \frac{x}{2} \cdot \sin hx}{\sqrt{3} - \sqrt{2 + \cos x}} \times \frac{\sqrt{3} + \sqrt{2 + \cos x}}{\sqrt{3} + \sqrt{2 + \cos x}}$$

$$= \lim_{x \to 0} \frac{2 \tan \frac{x}{2} \cdot \sin hx(\sqrt{3} + \sqrt{2 + \cos x})}{1 - \cos x}$$

$$= \lim_{x \to 0} \frac{2 \tan \frac{x}{2} \cdot \sin hx(\sqrt{3} + \sqrt{2 + \cos x})}{1 - \cos x}$$

$$= \lim_{x \to 0} \frac{\frac{2 \sin \frac{x}{2}}{\cos \frac{x}{2}} \cdot \sin hx(\sqrt{3} + \sqrt{2 + \cos x})}{2 \sin^2 \frac{x}{2}}$$

$$= \lim_{x \to 0} \frac{\sin hx(\sqrt{3} + \sqrt{2 + \cos x}}{\cos \frac{x}{2} \cdot \sin \frac{x}{2}}$$

$\because x \to 0 \Rightarrow \cos \frac{x}{2} \to 1$ and $\sin \frac{x}{2} \to \frac{x}{2}$

$\sin hx \to x$

$$= \frac{x(\sqrt{3} + \sqrt{2 + 1})}{\frac{x}{2}} = 2(\sqrt{3} + \sqrt{3})$$

$$= 2(2\sqrt{3}) = 4\sqrt{3}$$

Question14

The quadratic equation whose roots are $l = \lim_{\theta \to 0} \left(\frac{3 \sin \theta - 4 \sin^3 \theta}{\theta}\right)$ and $m = \lim_{\theta \to 0} \left(\frac{2 \tan \theta}{\theta(1 - \tan^2 \theta)}\right)$ is

AP EAPCET 2025 - 23rd May Morning Shift

Options:

A.

$x^2 + 5x + 6 = 0$

B.

$x^2 - 5x + 6 = 0$

C.

$x^2 - 5x - 6 = 0$

D.

$x^2 + 5x - 6 = 0$

Answer: B

Solution:

$l = \lim_{\theta \to 0} 3\left(\frac{\sin\theta}{\theta}\right) - \lim_{\theta \to 0} 4 \cdot \left(\frac{\sin\theta}{\theta}\right)^3 \cdot \theta^2 = 3 \times 1 - 4 \times 0 = 3$

$m = \lim_{\theta \to 0}\left(\frac{2\tan\theta}{\theta} \times \frac{1}{1-\tan^2\theta}\right) = 2 \times 1 \times \left(\frac{1}{1-0}\right) = 2$

$\therefore$ The quadratic equation whose roots are 3 and 2 is

$x^2 - 5x + 6 = 0.$

Question15

$$\lim_{x \to \infty} \frac{3x+4\cos^2 x}{\sqrt{x^2-5\sin^2 x}} =$$

AP EAPCET 2025 - 23rd May Morning Shift

Options:

A.

$\frac{3}{5}$

B.

$\frac{4}{5}$

C.

3

D.

1

Answer: C

Solution:

$$\text{Let } L=\lim_{x\to\infty}\frac{3x+4\cos^2 x}{\sqrt{x^2-5\sin^2 x}}$$

$$=\lim_{x\to\infty}\frac{x\left(3+\frac{4\cos^2 x}{x}\right)}{x\sqrt{1-5\left(\frac{\sin x}{x}\right)^2}}\left(\because \lim_{x\to\infty}\left(\frac{\sin x}{x}\right)^2=\lim_{y\to 0}\left(\frac{\sin\left(\frac{1}{y}\right)}{\frac{1}{y}}\right)\right)^2\Bigg)$$

$$=\lim_{y\to 0}\left(y\sin\left(\frac{1}{y}\right)\right)^2=(0\times a \text{ finite quantity })=0=\frac{3+0}{\sqrt{1-0}}=3$$

Question16

If a function,

$$f(x)=\begin{cases}\frac{\sqrt[3]{1+ax^2+bx^3}-\sqrt[3]{1-ax^2-bx^3}}{x^2}, & x<0\\ 5, & x=0\\ \frac{\tan 3x-\sin 3x}{bx^3}, & x>0\end{cases}$$

is continuous at $x=0$, then the geometric mean of a and b is

AP EAPCET 2025 - 23rd May Morning Shift

Options:

A.

$\frac{3}{2}$

B.

$\frac{9}{2}$

C.

$\frac{81}{4}$

D.

$\frac{9}{4}$

Answer: B

Solution:

$$=\lim_{x\to 0^-}\frac{\left(1+ax^2+bx^3\right)^{\frac{1}{3}}-\left(1-ax^2-bx^3\right)^{\frac{1}{3}}}{x^2}\left(\text{form }\frac{0}{0}\right)$$

$$=\lim_{x\to 0^-}\frac{\frac{1}{3}\left(1+ax+bx^3\right)^{\frac{1}{3}-1}\left(2ax+3bx^2\right)-\frac{1}{3}\left(1-ax^2-bx^3\right)^{\frac{1}{3}-1}(-2ax-3bx)}{2x}$$

(Using L Hospital rule)

$$=\frac{\frac{1}{3}\times(1+0+0)^{\frac{-2}{3}}(2a+0)-\frac{1}{3}(1-0-0)^{\frac{-2}{3}}(-2a-0)}{2}$$

(cancel x from both $N^r\times\mathrm{Dr}$)

$$=\frac{\frac{2a}{3}+\frac{2a}{3}}{2}=\frac{4a}{6}=\frac{2a}{3}\qquad\ldots(i)$$

RHL

$$\left(3x+\frac{(3x)^3}{3}+\frac{2(3x)^5}{15}+\ldots\right)$$

$$=\lim_{x\to 0^+}\frac{\tan 3x-\sin 3x}{bx^3}=\lim_{x\to 0^+}\frac{-\left(3x-\frac{(3x)^3}{3!}+\ldots\right)}{bx^3}$$

$$=\lim_{x\to 0^+}\frac{\frac{27x^3}{3}+\frac{27x^3}{6}+\text{(higher terms are zero)}}{bx^3}=\frac{27}{2b}$$

$f(0)=5$

$\because\quad f(x)$ is continuous at $x=0$.

$\therefore\quad \frac{2a}{3}=5=\frac{27}{2b}$

$\Rightarrow\quad a=\frac{15}{2},b=\frac{27}{10}$

$\therefore$ Geometric mean of a and $b=\sqrt{ab}=\sqrt{\frac{15}{2}\times\frac{27}{10}}=\frac{9}{2}$

Question17

$[x]$ denotes the greater integer less than or equal to x. If $\{x\}=x-[x]$ and $\lim\limits_{x\to 0}-\frac{\sin^{-1}(x+[x])}{2-\{x\}}=\theta$, then $\sin\theta+\cos\theta=$

Options:

A.

-1

B.

0

C.

1

D.

$\sqrt{2}$

Answer: A

Solution:

Evaluate $[x]$ as $x \to 0^-$

As x approaches 0 from the left $[x] = -1\{x\}$ as $x \to 0^-$

$\{x\} = x - [x] = x - (-1) = x + 1$

as $x \to 0^-, \{x\} \to 1$

$$\lim_{x \to 0^-} \frac{\sin^{-1}(x + [x])}{2 - \{x\}}$$
$$= \lim_{x \to 0^-} \frac{\sin^{-1}(x - 1)}{2 - (x + 1)}$$
$$= \lim_{x \to 0^-} \frac{\sin^{-1}(x - 1)}{1 - x}$$

as $x \to 0^-$, the limit becomes

$\frac{\sin^{-1}(-1)}{1-0} = -\frac{\pi}{2} = -\frac{\pi}{2}$

Therefore, $\theta = -\frac{\pi}{2}$

$$\sin\theta + \cos\theta = \sin\left(-\frac{\pi}{2}\right) + \cos\left(-\frac{\pi}{2}\right)$$
$$= -1$$
$$\sin\theta + \cos\theta = -1$$

Question18

$$\lim_{n\to\infty} \frac{1}{n^3} \sum_{k=1}^{n} k^2 x =$$

AP EAPCET 2025 - 22nd May Evening Shift

Options:

A.

x

B.

$\frac{x}{2}$

C.

$\frac{x}{3}$

D.

$\frac{x}{4}$

Answer: C

Solution:

$$\begin{aligned}
&\lim_{n\to\infty} \frac{1}{n^3} \sum_{k=1}^{n} k^2 x \\
&= \lim_{n\to\infty} \frac{1}{n^3} \frac{n(n+1)(2n+1)x}{6} \\
&= \lim_{n\to\infty} \frac{n \cdot n \cdot n \left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)x}{6n^3} \\
&= \frac{x}{6} \lim_{n\to\infty} \left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) \\
&= \frac{x}{6} \times 2 = \frac{x}{3}
\end{aligned}$$

Question19

Let $f : R \to R$ be defined by

$$f(x)=\begin{cases} a-\frac{\sin[x-1]}{x-1}, & \text{if } x>1 \\ 1, & \text{if } x=1 \\ b-\left[\frac{\sin[x-1]-[x-1]}{([x-1])^3}\right., & \text{if } x<1 \end{cases}$$

where $[t]$ denotes the greatest integer less than or equal to t. If f is continuous at $x = 1$, then $a + b =$

AP EAPCET 2025 - 22nd May Evening Shift

Options:

A.

0

B.

1

C.

2

D.

3

Answer: B

Solution:

Since, $f(x)$ is continuous at $x = 1$

$$\Rightarrow \quad \lim_{x\to 1^-} f(x) = f(1) = 1$$
$$= \lim_{x\to 1^+} f(x)$$

Now,

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} b - \left\{\frac{\sin[x-1]-[x-1]}{[x-1]^3}\right\}$$

Put $x-1=t$

$$= \lim_{x\to 1^-} b - \left\{\frac{\sin[t]-[t]}{[t]^3}\right\}$$
$$= b - \left\{\frac{\sin(-1)-(-1)}{(-1)^3}\right\}$$

$= b + (-\sin 1 + 1) = b - \sin 1 + 1$

and

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}\left(a - \frac{\sin[x-1]}{x-1}\right)$$

Put $x-1=t$

$$= \lim_{x\to 1^+}\left(a - \frac{\sin[t]}{t}\right) = a - 1$$

By Eq. (i), $a=2$ and $b - \sin l + 1 = 1$

$\Rightarrow \quad a = 2$ and $b = \sin 1$

$\Rightarrow \quad a + b = 2 + \sin 1$

Question20

$$\lim_{y\to 0}\frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4} =$$

AP EAPCET 2025 - 22nd May Morning Shift

Options:

A.

$\frac{1}{4\sqrt{2}}$

B.

$\frac{1}{2\sqrt{2}(1+\sqrt{2})}$

C.

$\frac{1}{2\sqrt{2}}$

D.

$\frac{1}{4\sqrt{2}(1+\sqrt{2})}$

Answer: A

Solution:

$\lim_{y \to 0} \frac{\left(1+\sqrt{1+y^4}\right)^{\frac{1}{2}}-\sqrt{2}}{y^4}$

Using L'Hospital's rule,

$$\lim_{y \to 0} \frac{\frac{1}{2}\left[1+\sqrt{1+y^4}\right]^{\frac{-1}{2}}\left(\frac{1}{2}\left(1+y^4\right)^{\frac{-1}{2}}4y^3\right)}{4y^3}$$

$$\Rightarrow \frac{1}{2}\frac{1}{(\sqrt{2})}\left(\frac{1}{2}\right)=\frac{1}{4\sqrt{2}}$$

Question21

If $\lim_{x \to 0} \frac{\cos 2x-\cos 4x}{1-\cos 2x}=k$, **then** $\lim_{x \to k} \frac{x^k-27}{x^{k+1}-81}=$

AP EAPCET 2025 - 22nd May Morning Shift

Options:

A.

0

B.

1

C.

$\frac{1}{2}$

D.

$\frac{1}{4}$

Answer: D

Solution:

$$\lim_{x\to 0}\frac{\cos 2x-\cos 4x}{1-\cos 2x}$$
$$\Rightarrow \quad \lim_{x\to 0}\frac{2\sin 3x\sin x}{2\sin^2 x}=k$$

$$\lim_{x\to 0}\frac{3\sin x-4\sin^3 x}{\sin x}=3=k$$
$$\text{Now, } \lim_{x\to 3}\left(\frac{x^3-27}{x^4-81}\right)\Rightarrow \lim_{x\to 3}\frac{3x^2}{4x^3}$$
$$=\frac{3(3)^2}{4(3)^3}=\frac{1}{4}$$

Question22

If the function $f(x)=\begin{cases}1+\cos x, x\le 0\\ a-x, 0<x\le 2\\ x^2-b^2, x>2\end{cases}$

is continuous everywhere, then $a^2+b^2=$

AP EAPCET 2025 - 22nd May Morning Shift

Options:

A.

4

B.

8

C.

6

D.

12

Answer: B

Solution:

$$f(x)=\begin{cases}1+\cos x, & x\leq 0\\ a-x, & 0<x\leq 2\\ x^2-b^2, & x>2\end{cases}$$

$$\lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x)$$

$$2=a\Rightarrow a=2$$

$$\lim_{x\to 2^-} f(x)=f(2)=\lim_{x\to 2^+} f(x)$$

$$\Rightarrow \quad a-2=4-b^2\Rightarrow b^2=4$$

$$a^2+b^2=8$$

Question23

$$\lim_{x\to-\infty}\frac{5x^3-x^2\sin 5x}{x\cos 4x+7|x|^3-4|x|+3}=$$

AP EAPCET 2025 - 21st May Evening Shift

Options:

A.

$\frac{5}{4}$

B.

$-\frac{5}{4}$

C.

$-\frac{5}{7}$

D.

$\frac{5}{7}$

Answer: C

Solution:

$\lim\limits_{x \to -\infty} \frac{5x^3 - x^2 \sin 5x}{x \cos 4x + 7|x|^3 - 4|x| + 3}$

Given, $x \to -\infty \Rightarrow \frac{1}{x} \to 0^-$

$\therefore \quad |x| = -x$

Now, dividing N^t and D^t by x^3, we get

$\lim\limits_{x \to -\infty} \frac{5 - \frac{\sin 5x}{x}}{\frac{\cos 4x}{x^2} - 7 + \frac{4}{x^2} + \frac{3}{x^3}}$

Clearly, $\lim\limits_{x \to -\infty} \frac{\sin 5x}{x} = 0$

and $\lim\limits_{x \to -\infty} \frac{\cos 4x}{x^2} = 0$

$$\therefore \lim_{x \to -\infty} \frac{5x^3 - x^2 \sin 5x}{x \cos 4x + 7|x|^3 - 4|x| + 3}$$
$$= \frac{5}{-7} = -\frac{5}{7}$$

Question24

If $\lim\limits_{x \to a^+} f(x) = p$, $\lim\limits_{x \to a^-} f(x) = m$ and $f(a) = k$, then which one of the following is true?

AP EAPCET 2025 - 21st May Evening Shift

Options:

A.

When $p - k \neq 0$ and $m - k \neq 0$, then $f(x)$ is continuous at $x = a$

B.

When $p - k = 0$ and $m - k \neq 0$, then $f(x)$ is left continuous at $x = a$

C.

When $p - k \neq 0$ and $m - k = 0$, then $f(x)$ is right continuous at $x = a$

D.

When $p-m=0$ and $p-k=0$, then $f(x)$ is right continuous at $x=a$

Answer: D

Solution:

$\lim_{x \to a^+} f(x)=p, \lim_{x \to a^-} f(x)=m$

and $\quad f(a)=k$

$$\text{LHL}=\text{RHL}=f(a)$$
$$\therefore \quad p=m=k$$

$p-m=0$ and $p-k=0$, then $f(x)$ is right continuous at $x=a$.

Question25

If a function f defined by

$$f(x)=\begin{cases} \frac{1-\cos 4x}{x^2}, & x<0 \\ \frac{a}{\sqrt{x}}, & x=0 \\ \frac{\sqrt{16+\sqrt{x}-4}}{\sqrt{16+0}} \end{cases}$$

is continuous at $x=0$, then $a=$

AP EAPCET 2025 - 21st May Evening Shift

Options:

A.

8

B.

4

C.

3

D.

2

Answer: A

Solution:

$$
\begin{aligned}
&\text{We have, } f(0)=a \\
&\text{LHL } = \lim_{x \to a^-} f(x) = \lim_{x \to 0} \frac{1-\cos 4x}{x^2} \times \frac{16}{16} \\
&= \lim_{x \to 0} \frac{1-\cos 4x}{(4x)^2} \times 16 = \frac{1}{2} \times 16 = 8 \\
&\text{RHL } = \lim_{x \to 0^+} f(x) = \lim_{x \to 0} \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} \\
&= \lim_{x \to 0} \frac{\sqrt{x}[\sqrt{16+\sqrt{x}}+4]}{16+\sqrt{x}-16} \\
&= \lim_{x \to 0} \sqrt{16+\sqrt{x}}+4 = 8 \\
&\therefore \quad a = 8
\end{aligned}
$$

Question26

$$\lim_{x \to \infty} \frac{(\sqrt{2})-\sqrt{1+\cos x}}{\sqrt{15+\cos 2x-4}} =$$

AP EAPCET 2025 - 21st May Morning Shift

Options:

A.

$-\frac{1}{\sqrt{2}}$

B.

$\frac{1}{\sqrt{2}}$

C.

$\sqrt{2}$

D.

$-\sqrt{2}$

Answer: A

Solution:

$\lim_{x\to\infty} \frac{\sqrt{2}-\sqrt{1+\cos x}}{\sqrt{15+\cos 2x-4}}$

By rationalizing N^r and D^r, we get

$$\lim_{x\to0}\frac{2-1-\cos x}{\sqrt{2}+\sqrt{1+\cos x}}\times\frac{\sqrt{15+\cos 2x}+4}{15+\cos 2x-16}$$
$$=\lim_{x\to0}\frac{1-\cos x}{-(1-\cos 2x)}\times\frac{4+4}{\sqrt{2}+\sqrt{2}}$$
$$=-\frac{8}{2\sqrt{2}}\lim_{x\to0}\frac{\frac{1-\cos x}{x^2}\times x^2}{1-\frac{\cos 2x}{(2x)^2}\times 4x^2}$$

Using result $\lim_{x\to0}\frac{1-\cos x}{x^2}=\frac{1}{2}$

$=-\frac{8}{2\sqrt{2}}\times\frac{\frac{1}{2}}{\frac{1}{2}}\times\frac{1}{4}=-\frac{1}{\sqrt{2}}$

Question27

If a real valued function

$$f(x)=\begin{cases}\frac{x^2+(a+3)x+(a+1)}{x+3} & \text{, when } x\neq-3\\ -\frac{5}{2} & \text{, when } x=-3\end{cases}$$

is continuous at $x=-3$, then $\lim_{x\to a}\left(x^2+x+1\right)=$

AP EAPCET 2025 - 21st May Morning Shift

Options:

A.

$\frac{7}{4}$

B.

$\frac{5}{2}$

C.

$\frac{4}{7}$

D.

$\frac{2}{5}$

Answer: A

Solution:

$\lim_{x \to 3} \frac{x^2+(a+3)x+(a+1)}{x+3}$

It is $\frac{0}{0}$ form

$\lim_{x \to 3} \frac{2x+(a+3)}{1} = -6 + a + 3 = a - 3$

$\because f(x)$ is continuous $a - 3 = -\frac{5}{2}$

$$a = 3 - \frac{5}{2} = \frac{1}{2}$$
$$\lim_{x \to \frac{1}{2}} \left(x^2 + x + 1\right) = \frac{1}{4} + \frac{1}{2} + 1 = \frac{1}{4} + \frac{3}{2}$$
$$= \frac{1+6}{4} = \frac{7}{4}$$

Question28

$$\lim_{x \to 0} \frac{x \tan 2x - 2x \tan x}{(1-\cos 3x)(\operatorname{cosec} x - \cot x)^2} =$$

AP EAPCET 2025 - 21st May Morning Shift

Options:

A.

$\frac{4}{9}$

B.

$\frac{8}{9}$

C.

$\frac{16}{9}$

D.

$\frac{32}{9}$

Answer: C

Solution:

$$\lim_{x \to 0} \frac{x \tan 2x - 2x \tan x}{(1 - \cos 3x)(\operatorname{cosec} x - \cot x)^2}$$

$$= \lim_{x \to 0} \frac{2x \tan x \left[\frac{1}{1-\tan^2 x} - 1\right]}{\frac{1-\cos 3x}{(3x)^2} \times (3x)^2 \left[\frac{1-\cos x}{\sin x}\right]^2}$$

$$= \lim_{x \to 0} \frac{2x \tan x \left(1 - 1 + \tan^2 x\right) \times \sin^2 x}{\left(1 - \tan^2 x\right) \frac{1-\cos 3x}{(3x)^2}}$$

$$\times 9x^2 \frac{(1 - \cos x)^2}{x^2} \times x^4$$

$$= \lim_{x \to 0} \frac{2}{9} \times \frac{\tan x}{x} \times \frac{\tan^2 x}{\frac{1}{2}} \times \frac{\sin^2 x}{x^4 \times \left(\frac{1}{2}\right)^2}$$

$$= \lim_{x \to 0} \frac{2}{9} \times 2 \times 4 = \frac{16}{9}$$

Question29

Match the functions in Column I with their properties in Column II. In the following [x] denotes the greatest integer less than or equal to x.

Column I		Column II	
A	$x\|x\|$	I	Strictly increasing and continuous in $(-1, 1)$
B	$\sqrt{\|x\|}$	II	Continuous but not differentiable in $(-1, 1)$
C	$x + [x]$	III	Differentiable in $(-1, 1)$

Column I		Column II	
D	$\|x-1\|+\|x+1\|+\|x\|$	IV	Differentiable in $(-1,0)\cup(0,1)$
		V	Strictly increasing and not differentiable in $(-1,1)$

The correct match is

AP EAPCET 2025 - 21st May Morning Shift

Options:

A.

A-III, B-V, C-II, D-I

B.

A-II, B-III, C-I, D-V

C.

A-I, B-II, C-V, D-IV

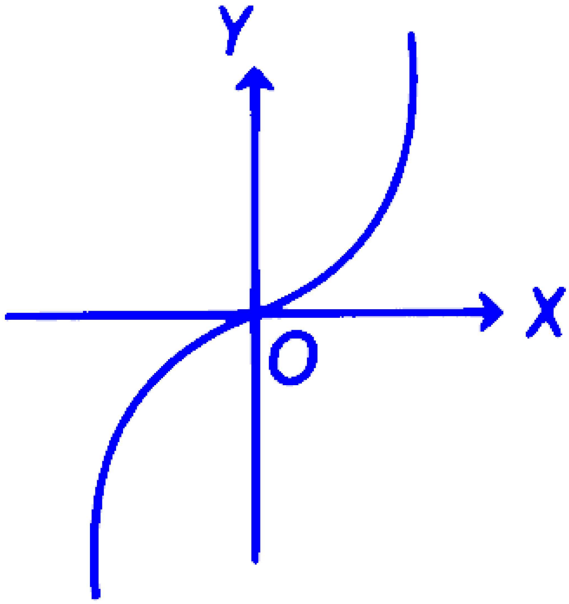
D.

A-IV, B-I, C-V, D-III

Answer: C

Solution:

Which is continuous and strictly increasing



(B) $f(x)=\sqrt{|x|}=\begin{cases}\sqrt{x}, x\geq 0\\ \sqrt{-x}, x<0\end{cases} f'(x)=\frac{1}{2\sqrt{x}}(\pm 1)$ Which is not defined at $x=0$, this is continuous but not differentiable at $x=0$

(C) $f(x)=x+[x]=\begin{cases}x-1, & -1\leq x<0\\ x, & 0\leq x<1\\ x+1, & 1\leq x<2\end{cases}$

This is strictly increasing but not differentiable at integer.

$(D) f(x)=|x-1|+|x|+|x+1|$

$$=\begin{cases}-3x, & -1\leq x<0\\ 2-x, & 0\leq x<1\\ 3x, & 1\leq x<\infty\end{cases}$$

Clearly, it is not differentiable at $x=-1$, 0,1 but differentiable in $(-1,0)\cup(0,1)$

A-I, B-II, C-V, D-IV

Question30

Consider the following functions

I. $f(x)=\begin{cases}\frac{1}{2}-x & , x<\frac{1}{2}\\ \left(\frac{1}{2}-x\right)^2 & , x\geq\frac{1}{2}\end{cases}$

II. $f(x)=|3x-1|$

III. $f(x) = x|x|$

IV. $f(x) = |x|$

Then, on $[0, 1]$ Lagrange's mean value theorem is applicable to the functions

AP EAPCET 2025 - 21st May Morning Shift

Options:

A.

III, IV

B.

II, III

C.

I, III

D.

II, IV

Answer: A

Solution:

(I) $f(x) = \begin{cases} \frac{1}{2} - x, & x < \frac{1}{2} \\ \left(\frac{1}{2} - x\right)^2 & x \geq \frac{1}{2} \end{cases}$

$f(x)$ is not differentiable at $x = \frac{1}{2}$

LMVT not applicable

(II) $f(x) = |3x - 1|$

$f(x)$ is not differentiable at $x = \frac{1}{3}$

$\therefore$ LMVT is not applicable.

(III) $f(x) = x|x|$ it is continuous and differentiable in $(0, 1)$

= LMVT applicable.

(V) $f(x) = |x|$ it is continuous and differentiable in $(0, 1)$

$\therefore$ LMVT is applicable.

Question31

$$\lim_{x \to \infty} \left[\frac{n+1}{n^2+1^2} + \frac{n+2}{n^2+2^2} + \frac{n+3}{n^2+3^2} + \ldots + \frac{n+2n}{n^2+4n^2} \right] =$$

AP EAPCET 2025 - 21st May Morning Shift

Options:

A.

$\tan^{-1} 2 + \frac{1}{2} \log 3$

B.

$\frac{\pi}{4} + \frac{1}{2} \log 3$

C.

$\tan^{-1} 2 + \frac{1}{2} \log 5$

D.

$\frac{\pi}{4} + \frac{1}{2} \log 5$

Answer: C

Solution:

$$\lim_{n \to \infty} \sum_{r=1}^{2n} \frac{n+r}{n^2+(r)^2}$$
$$\lim_{n \to \infty} \frac{1}{n} \sum_{r=1}^{2n} \frac{1+\frac{r}{n}}{1+\left(\frac{r}{n}\right)^2}$$
$$= \int_0^2 \frac{1+x}{1+(x)^2} dx$$
$$= \int_0^2 \frac{1}{1+x^2} dx + \int_0^2 \frac{x}{1+x^2} dx$$
$$= \left(\tan^{-1} x\right)_0^2 + \left(\frac{1}{2} \ln\left(1+x^2\right)\right)_0^2$$
$$= \tan^{-1} 2 + \frac{1}{2} \ln 5$$

Question32

Let $f(x) = \begin{cases} 1+\frac{2x}{a}, & 0 \leq x \leq 1 \\ ax, & 1 < x \leq 2 \end{cases}$.If $\lim_{x \to 1} f(x)$ exists, then the sum of the cubes of the possible values of a is

AP EAPCET 2024 - 23th May Morning Shift

Options:

A. 1

B. 5

C. 9

D. 7

Answer: D

Solution:

$\because \lim_{x \to 1} f(x)$ exists

$\therefore \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$

$Now, \lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1-h)$

$= \lim_{h \to 0} \left(1 + \frac{2(1-h)}{a}\right) = 1 + \frac{2}{a}$

and $\lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(1+h)$

$= \lim_{h \to 0} a(1+h) = a$

$\therefore \quad 1 + \frac{2}{a} = a$

$\Rightarrow \quad a + 2 = a^2$

$\Rightarrow \quad a^2 - a - 2 = 0$

$\Rightarrow \quad a^2 - 2a + a - 2 = 0$

$\Rightarrow \quad a(a-2) + 1(a-2) = 0$

$\Rightarrow \quad (a-2)(a+1) = 0$

$\Rightarrow \quad a = -1, 2$

$\therefore$ Sum of the cubes of the possible values of a is $-1 + 8 = 7$.

Question33

Let $[P]$ denote the greatest integer $\leq P$. If $0 \leq a \leq 2$, then the number of integral values of ' a ' such that $\lim_{x \to a} \left(\left[x^2\right] - [x]^2\right)$ does not exist is

AP EAPCET 2024 - 23th May Morning Shift

Options:

A. 3

B. 2

C. 1

D. 0

Answer: B

Solution:

At $x = 0$

$\mathrm{LHL} = \lim\limits_{x \to 0^-} \left(\left[x^2\right] - [x]^2 \right) = 0 - 1 = -1$
$\mathrm{RHL} = \lim\limits_{x \to 0^+} \left(\left[x^2\right] - [x]^2 \right) = 0 - 0 = 0$

$\therefore$ LHL $\neq$ RHL

Limit does not exists at $x = 0$.

At $x = 1$

$\mathrm{LHL} = \lim_{x \to 1^-} \left(\left[x^2\right] - [x]^2 \right) = 0 - 0 = 0$

$\mathrm{RHL} = \lim_{x \to 1^+} \left(\left[x^2\right] - [x]^2 \right) = 1 - 1 = 0$

As LHL $=$ RHL, so limit exits at $x = 1$

At $x = 2$

$\mathrm{LHL} = \lim\limits_{x \to 2^-} \left(\left[x^2\right] - [x]^2 \right) = 3 - 1 = 2$
$\mathrm{RHL} = \lim\limits_{x \to 2^+} \left(\left[x^2\right] - [x]^2 \right) = 4 - 4 = 0$

As LHL $\neq$ RHL, so limit does not exists. Hence, there are two integral points where the limits does not exists.

Question34

If $f(x) = \begin{cases} \frac{\sqrt{a^2 - ax + x^2} - \sqrt{x^2 + ax + a^2}}{\sqrt{a + x} - \sqrt{a - x}}, & x \neq 0 \text{ is} \\ K & x = 0 \end{cases}$ continuous at $x = 0$, then K is equal to

AP EAPCET 2024 - 23th May Morning Shift

Options:

A. $-\sqrt{a}$

B. $\sqrt{a}$

C. -1

D. $a + \sqrt{a}$

Answer: A

Solution:

As $f(x)$ is continuous at $x=0$
$f(0)=\lim_{x \to 0} f(x)$

$$K=\lim_{x \to 0} \frac{\sqrt{a^2-ax+x^2}-\sqrt{x^2+ax+a^2}}{\sqrt{a+x}-\sqrt{a-x}}$$

$$\Rightarrow K=\lim_{x \to 0}$$

$$\frac{\sqrt{a^2-ax+x^2}-\sqrt{x^2+ax+a^2}}{\sqrt{a+x}-\sqrt{a-x}} \times \frac{\sqrt{a^2-ax+x^2}+\sqrt{x^2+ax+a^2}}{\sqrt{a^2-ax+x^2}+\sqrt{x^2+ax+a^2})} \times \frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}+\sqrt{a-x}}$$

$$=\lim_{x \to 0} \frac{-2ax(\sqrt{a+x}+\sqrt{a-x})}{2x\left(\sqrt{x^2-ax+a^2}+\sqrt{x^2+ax+a^2}\right)}$$

$$=\frac{-a(2\sqrt{a})}{2a}=-\sqrt{a}$$

$\therefore \quad K=-\sqrt{a}$

Question35

If $f(x)=\begin{cases} ax^2+bx-\frac{13}{8}, & x \leq 1 \\ 3x-3, & 1<x \leq 2 \\ bx^3+1, & x>2 \end{cases}$ **is differentiable** $\forall x \in R$**, then** $a-b$ **is equal to**

AP EAPCET 2024 - 23th May Morning Shift

Options:

A. $\frac{9}{8}$

B. $\frac{5}{4}$

C. $\frac{11}{8}$

D. $\frac{1}{4}$

Answer: A

Solution:

$$f(x)=\begin{cases}ax^2+bx-\frac{13}{8}, & x\le 1\\ 3x-3, & 1<x\le 2\\ bx^3+1, & x>2\end{cases}$$

$$f'(x)=\begin{cases}2ax+b, & x\le 1\\ 3, & 1<x\le 2\\ 3bx^2, & x>2\end{cases}$$

As, the function is differentiable at $x \in R$,

$$\lim_{x\to 1} 2ax+b=3 \text{ and } \lim_{x\to 2} 3bx^2=3$$
$$\Rightarrow 2a+b=3 \text{ and } 12b=3 \Rightarrow b=\frac{1}{4}$$
$$2a=3-\frac{1}{4}\Rightarrow 2a=\frac{11}{4}\Rightarrow a=\frac{11}{8}$$
$$\therefore \quad a-b=\frac{11}{8}-\frac{1}{4}=\frac{9}{8}$$

Question36

In each of the following options, a function and an interval are given. Choose the option containing the function and the interval for which Lagrange's mean value theorem is not applicable

AP EAPCET 2024 - 23th May Morning Shift

Options:

A. $f(x)=|x|, 1\le x\le 5$

B. $f(x)=[x], [\sqrt{2}, \sqrt{3}]$

C. $f(x)=\log\left(x^2-1\right), \left[\frac{1}{e}, e-2\right]$

D. $f(x)=e^x, [-e, e]$

Answer: C

Solution:

For option (a), $f(x)=|x|=x$ as $1\le x\le 5$

As $f(x)$ is a polynomial function in its domain, so it is continuous and differentiable.

$f'(x) = 1, f(1) = 1$ and $f(3) = 5$

$f'(x) = \dfrac{f(5) - f(1)}{5 - 1} = 1 \in [1, 5]$

$\therefore$ Lagrange's mean value theorem is applicable.

For option (b)

$f(x) = [x] = 1$ for $x \in [\sqrt{2}, \sqrt{3}]$

As $f(x)$ is a constant function, so Lagranye's mean value theorem is applicable.

For option (c),

The function $f(x) = x^2 - 1$ is continuous if $x^2 - 1 > 0$ i.e. either $x < -1$ or $x > 1$.

For $x = \frac{1}{e}, x^2 - 1 > 0$, but $\frac{1}{e^2} - 1 < 0$

So, the value is not within the domain Thus, lagrange mean value theorem is not applicable.

For option (d), $f(x) = e^x$ is continuous and differentiable over $[-e, e]$

$f'(x) = e^x$ and $f(-e) = \frac{1}{e^i} f(l) = e^e$

$$
\begin{aligned}
f'(e) &= \frac{f(b) - f(a)}{b - a} = \frac{e^e - \frac{1}{e^e}}{e + e} \\
&= \frac{(e^e)^2 - 1}{e^e \times 2e} \\
&= \frac{e^{e-1}}{2} - \frac{1}{2e^{e+1}} \in [-e, e]
\end{aligned}
$$

$\therefore$ Lagrange mean value theorem is applicable.

Question37

The function $f(x) = \begin{cases} \frac{x - |x|}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$

AP EAPCET 2024 - 23th May Morning Shift

Options:

A. is continuous, $\forall x \in R$

B. has maximum value 2

C. has neither minimum nor maximum

D. has minimum value 2

Answer: B

Solution:

$$f(x)=\begin{cases}\frac{x-|x|}{x}, & x\neq 0\\ 2, & x=0\end{cases}$$
$$=\begin{cases}2, & x<0\\ 0, & x>0\\ 2, & x=0\end{cases}$$
$$\left[\because |x|=\begin{cases}x, & \text{if } x>0\\ -x, & \text{if } x<1\end{cases}\right.$$

$\therefore f(x)$ has maximum value 2 and minimum value 0 .

Question38

$\lim\limits_{x\to\infty}\frac{[2x-3]}{x}$ is equal to

AP EAPCET 2024 - 22th May Evening Shift

Options:

A. 0

B. ∞

C. -3

D. 2

Answer: D

Solution:

$$\lim_{x\to\infty}\frac{[2x-3]}{x}$$

$$\Rightarrow \lim_{x\to\infty}\left(\frac{2x}{x}-\frac{3}{x}\right)\Rightarrow 2-\lim_{x\to\infty}\frac{3}{x}$$
$$\Rightarrow 2-0\Rightarrow 2$$

Question39

$\lim_{x \to 0} \frac{\cos 2x - \cos 3x}{4x - \cos 5x}$ **is equal to** $\cos 4x - \cos 5x$

AP EAPCET 2024 - 22th May Evening Shift

Options:

A. $\frac{5}{9}$

B. 1

C. $\frac{3}{4}$

D. $\frac{2}{5}$

Answer: A

Solution:

$\lim_{x \to 0} \frac{\cos 2x - \cos 3x}{\cos 4x - \cos 5x} \quad \left(\frac{0}{0} \text{ form }\right)$

By L' Hospital rule,

$= \lim_{x \to 0} \frac{-2 \sin 2x + 3 \sin 3x}{-4 \sin 4x + 5 \sin 5x} \quad \left(\frac{0}{0} \text{ form }\right)$

Again by L' Hospital rule

$$= \lim_{x \to 0} \frac{-4 \cos 2x + 9 \cos 3x}{-16 \cos 4x + 25 \cos 5x}$$
$$= \frac{-4 \times 1 + 9 \times 1}{-16 \times 1 + 25 \times 1} = \frac{5}{9}$$

Question40

If a real valued function $f(x) = \begin{cases} \frac{2x^2 + (k+2)x + 9}{3x^2 - 7x - 6}, & \text{for } x \neq 3 \\ 1, & \text{for } x = 3 \end{cases}$ **is continuous at** $x = 3$ **and** l **is a finite value, then** $l - k$ **is equal to**

Options:

A. $\frac{31}{11}$

B. $\frac{124}{11}$

C. 24

D. 32

Answer: B

Solution:

Given that

$$f(x)=\begin{cases}\frac{2x^2+(k+2)x+9}{3x^2-7x-6} & \text{for } x\neq 3\\ l & \text{for } x=3\end{cases}$$

is continuous at $x=3$

$$\Rightarrow \text{RHL} = \text{LHL} = \text{Value of function}$$
$$\therefore \lim_{x\to 3^+}\frac{2x^2+(k+2)x+9}{3x^2-7x-6} = \text{Value of}$$

function at $x=3$

$$\Rightarrow \lim_{x\to 3^+}\frac{2x^2+(k+2)x+9}{3x^2-7x-6}=l$$
$$\Rightarrow \lim_{x\to 3^+}\frac{2x^2+(k+2)x+9}{(3x+2)(x-3)}=l$$

For limit to be exist, numerator also have a factor $(x-3)$ so

$2x^2+(k+2)x+9=(x-3)(2x+a)$

Expand and equate the coefficients

$k+2=a-6$
$9=-3a\Rightarrow a=-3\Rightarrow k=-11$

So, $2x^2+(k+2)x+9=(x-3)(2x-3)$

Then, $\lim_{x\to 3^+}\frac{(x-3)(2x-3)}{(3x+2)(x-3)}=1$

$\Rightarrow \quad \frac{6-3}{9+2}=l\Rightarrow l=\frac{3}{11}$

So, $l-k=\frac{3}{11}+11=\frac{124}{11}$

Question41

$$\lim_{x \to o} \left[\frac{1}{x} - \frac{1}{e^x - 1}\right] =$$

AP EAPCET 2024 - 22th May Morning Shift

Options:

A. 0

B. 1

C. 2

D. 1/2

Answer: D

Solution:

$\lim_{x \to o} \left[\frac{1}{x} - \frac{1}{e^x - 1}\right] = \lim_{x \to o} \frac{e^x - 1 - x}{x(e^x - 1)}$

Using L'Hospital's rule

$= \lim_{x \to o} \frac{e^x - 1}{e^x - 1 + xe^x}$

Again, using L'Hospital rule, we get

$= \lim_{x \to o} \frac{e^x}{e^x - e^x + xe^x} = \frac{e^o}{2e^o + 0} = \frac{1}{2}.$

Question42

Let $f(x) = \begin{cases} 0, & x = 0 \\ 2 - x, & \text{for } 0 < x < 1 \\ 2, & \text{for } x = 1 \\ \frac{1}{2} - x, & \text{for } 1 < x < 2 \\ \frac{-3}{2}, & \text{for } x \geq 2 \end{cases}$

then which of the following is true

AP EAPCET 2024 - 22th May Morning Shift

Options:

A.

f is right continuous at $x = 0$

B.

f is left continuous at $x = 1$

C.

f is right continuous at $x = 1$

D.

f is continuous at $x = 2$

Answer: D

Solution:

$$f(x) = \begin{cases} 0, & x = 0 \\ 2 - x, & \text{for } 0 < x < 1 \\ 2, & \text{for } x = 1 \\ \frac{1}{2} - x, & \text{for } 1 < x < 2 \\ \frac{-3}{2}, & \text{for } x \geq 2 \end{cases}$$

continuity at $x = 1$

As x approaches 1 from the left

$\lim_{x \to 1^+} f(x) = \frac{1}{2} - 1 = -\frac{1}{2}$

The value of $f(x)$ at $x = 1$

$f(1) = 2$

LHL $\neq$ RHL

Therefore, $f(x)$ is not continuous at $x = 1$

Continuity at $x = 2$

As x approaches 2 from the left (1 < x < 2)

$\lim_{x \to 2^+} f(x) = \frac{1}{2} - 2 = -\frac{3}{2}$

The value of $f(x)$ at $x = 2$

$f(x) = -\frac{3}{2}$

The LHL and actual value of the function at $x = 2$ are equal to RHL

$\lim_{x \to 2^+} f(x) = f(2) = -\frac{3}{2}$

Therefore, $f(x)$ is continuous at $x = 2$

Question43

If $f(x) = \left(\frac{1+x}{1-x}\right)^{\frac{1}{x}}$ is continuous at $x = 0$, then $f(0) =$

AP EAPCET 2024 - 22th May Morning Shift

Options:

A. $e^{\frac{1}{2}}$

B. e^2

C. e^{-2}

D. $e^{\frac{-1}{2}}$

Answer: B

Solution:

$f(x) = \left(\frac{1+x}{1-x}\right)^{1/x}$

We will calculate the limit of this expression as x approaches 0 .

Let $L = \left(\frac{1+x}{1-x}\right)^{1/x}$

$L = \lim_{x \to 0} \frac{1}{x} \ln \left(\frac{1+x}{1-x}\right)^{1/x} \ldots$ (i)(natural log)

Let $y = \log \frac{1+x}{1-x}$

$= \log(1+x) - \log(1-x)$

Using Taylor's expansions for $\log(1+x)$ and $\log(1-x)$ around $x = 0$

$\log(1+x) \approx x$

$\log(1-x) \approx -x$

Then, $y = x - (-x)$

$y = 2x \Rightarrow \log\left(\frac{1+x}{1-x}\right) = 2x \ldots$ Eq.

From Eq. (i), we get

$$\begin{aligned} \log L &= \lim_{x \to 0} \frac{1}{x} \ln\left(\frac{1+x}{1-x}\right) \\ &= \lim_{x \to 0} \frac{1}{x} \times 2x \text{ using} \\ \log L &= \lim_{x \to 0} 2 \\ \log L &= 2 \\ f(x) &= L = e^2 \end{aligned}$$

$\because f(x)$ is continuous at, $x = 0, f(0) = e^2$

Question44

The function $f(x) = |x - 24|$ is

AP EAPCET 2024 - 22th May Morning Shift

Options:

A. Differentiable on $[0, 25]$

B. not continuous at $x = 24$

C. neither continuous nor differentiable on $[0, 25]$

D. Continuous on $[0, 25]$ but not differentiable on $[0, 25]$

Answer: D

Solution:

The function $f(x) = |x - 24|$ is being evaluated for its continuity and differentiability on the interval $[0, 25]$.

Continuity

The function $f(x) = |x - 24|$ is continuous over the interval $[0, 25]$. The absolute value function is continuous across its domain, including the point $x = 24$ within this interval.

Differentiability

The function $f(x) = |x - 24|$ is differentiable on the interval $(0, 25)$ except at the point $x = 24$. At $x = 24$, the left-hand derivative is $f'(24) = -1$ and the right-hand derivative is $f'(24) = 1$. Since the left-hand and right-hand derivatives at $x = 24$ are not equal, the function is not differentiable at this point.

For the subintervals $0 < x < 24$ and $24 < x \leq 25$, the derivative of $f(x)$ is well defined.

Conclusion

Thus, the function $f(x) = |x - 24|$ is continuous on $[0, 25]$ but not differentiable on $[0, 25]$, specifically due to the point $x = 24$.

Question45

$$\lim_{n\to\infty}\left(\frac{1}{\sqrt{n^2}}+\frac{1}{\sqrt{n^2-1}}+\ldots+\frac{1}{\sqrt{n^2-(n-1)^2}}\right)=$$

AP EAPCET 2024 - 22th May Morning Shift

Options:

A. $2\sqrt{\pi}$

B. $\frac{2}{\sqrt{\pi}}$

C. $\frac{\pi}{2}$

D. $\frac{3\pi}{2}$ (-)$(\because n = \infty)$

Answer: C

Solution:

$$\lim_{n\to\infty}\left(\frac{1}{\sqrt{n^2}}+\frac{1}{\sqrt{n^2-1}}+\frac{1}{\sqrt{n^2-(n-1)^2}}\right)$$

We can observe that,

$$T_n = \frac{1}{\sqrt{n^2 - r^2}}$$

$$\Rightarrow \lim_{n\to\infty}\sum_{n=0}^{n-1}\frac{1}{\sqrt{n^2-r^2}} = \lim_{n\to\infty}\sum_{n=0}^{n-1}\frac{1}{n\sqrt{1-\left(\frac{r}{n}\right)^1}}$$

Let $\frac{r}{n} = x$

$\frac{1}{n} = dx$

limit $r = 0 \Rightarrow x = 0$

$r = n - 1$

$\Rightarrow \quad x = 1 - \frac{1}{n}$

$x = 1 \qquad (\because n = \infty)$

$\Rightarrow \int_0^1 \frac{dx}{\sqrt{1-x^2}}$

$= \left(\sin^{-1}\right)_0^1$

$= \sin^{-1}(1) - \sin^{-1}(0)$

$= \frac{\pi}{2} - 0 = \frac{\pi}{2}$

Question46

$$\lim_{x \to 0} \left(\frac{\sin(\pi \cos^2 x}{x^2} \right) =$$

AP EAPCET 2024 - 21th May Evening Shift

Options:

A. $-\pi$

B. π

C. $\frac{\pi}{2}$

D. 1

Answer: B

Solution:

We have,

$$\Rightarrow \lim_{x \to 0} \frac{\sin\left(\pi \cos^2 x\right)}{x^2} = \lim_{x \to 0} \frac{\sin\left(\pi\left(1 - \sin^2 x\right)\right)}{x^2}$$

$\left[\because \sin^2 x + \cos^2 x = 1\right]$

$$= \lim_{x \to 0} \frac{\sin\left(\pi - \pi \sin^2 x\right)}{x^2} = \lim_{x \to 0} \frac{\sin\left(\pi \sin^2 x\right)}{x^2}$$

$$= \lim_{x \to 0} \frac{\sin\left(\pi \sin^2 x\right)}{\left(\pi \sin^2 x\right)} \times \frac{\left(\pi \sin^2 x\right)}{\left(x^2\right)}$$

$$= \lim_{x \to 0} \frac{\sin\left(\pi \sin^2 x\right)}{\left(\pi \sin^2 x\right)} \cdot \lim_{x \to 0} \frac{\pi \sin^2 x}{x^2}$$

$$= \pi \quad \left\{\because \lim_{x \to 0} \frac{\sin \theta}{\theta} = 1\right\}$$

Question47

$$\lim_{x \to 1} \left(\frac{x+x^2+x^3+\ldots+x^n-n}{x-1}\right) =$$

AP EAPCET 2024 - 21th May Evening Shift

Options:

A. $\frac{n(n+1)}{2}$

B. $\frac{n+1}{2}$

C. $\frac{2}{n}$

D. n

Answer: A

Solution:

we have

$$\lim_{x \to 1} \left(\frac{x + x^2 + x^3 + \ldots + x^n - n}{x - 1}\right)$$

$$= \lim_{x \to 1} \left[\frac{x + x^2 + x^3 + \ldots + x^n - n}{(x - 1)}\right]$$

$$= \lim_{x \to 1} \left[\frac{\begin{array}{c} x + x^2 + x^3 + \ldots + x^n \\ -(1 + 1 + \ldots + 1 \ldots n \text{ times }) \end{array}}{(x - 1)}\right]$$

$$= \lim_{x \to 1} \left[\frac{(x - 1) + \left(x^2 - 1\right) + \left(x^3 - 1\right) + \ldots + \left(x^n - 1\right)}{(x - 1)}\right]$$

$$= \lim_{x \to 1}\left[\frac{(x-1)+(x^2-1^2)+(x^3-1^3)+\ldots+(x^n-1^n}{(x-1)}\right]$$

So, divide each term is square bracket by $(x-1)$ as given in denominator and applying the limit, we get

$\lim_{x \to 1}\left[\frac{(x-1)+(x^2-1^2)+(x^3-1^3)+\ldots+(x^n-1^n)}{(x-1)}\right]$

$$= \underline{1(1)^0 + 2(1)^1 + 3(l)^2 + 4(1)^3 + \ldots + n(1)^{n-1}}$$

$$\left[\text{Formula used, } \lim_{x \to a}\frac{x^n - a^n}{x - a} = na^{n-1}\right]$$

$$= 1 + 2 + 3 + 4 + \ldots + n$$
$$= \frac{n(n+1)}{2}$$

Question48

If the function $f(x) = \frac{\sqrt{1+x}-1}{x}$ is continuous at $x = 0$, then $f(0) =$

AP EAPCET 2024 - 21th May Evening Shift

Options:

A. $-\frac{1}{2}$

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $-\frac{1}{3}$

Answer: C

Solution:

Given that the functions is continuous

$$\Rightarrow \lim_{x \Rightarrow 0} f(x) = f(0)$$
$$\Rightarrow \lim_{x \to 0}\frac{\sqrt{1+x}-1}{x} = \frac{\sqrt{1+0}-1}{0} = \frac{0}{0}$$
$$\left[\text{which is } \frac{0}{0} \text{ from}\right]$$

So, we apply L-Hospital rule

$$= \lim_{x \to 0} \frac{\frac{d}{dx}(\sqrt{1+x} - 1)}{\frac{d}{dx} x} = \lim_{x \to 0} \frac{\frac{1}{2}\sqrt{1+x}}{1}$$

$$= \lim_{x \to 0} \frac{1}{2\sqrt{1+x}}$$

On applying the limit, we get

$$\lim_{x \to 0} \frac{1}{2\sqrt{1+h}} = \frac{1}{2}$$

$$\therefore \quad f(0) = \frac{1}{2}$$

Question49

If $f(x) = \frac{5x \cdot \operatorname{cosec}(\sqrt{x}) - 1}{(x-2)\operatorname{cosec}(\sqrt{x})}$, then $\lim_{x \to \infty} f\left(x^2\right) =$

AP EAPCET 2024 - 21th May Morning Shift

Options:

A. 1

B. -1

C. 5

D. -5

Answer: C

Solution:

$$\text{If } f(x) = \frac{5x \cdot \operatorname{cosec}(\sqrt{x}) - 1}{(x-2)\operatorname{cosec}(\sqrt{x})}$$

$$\text{then, } \lim_{x \to \infty} f\left(x^2\right) = \ ?$$

$$f\left(x^2\right) = \frac{5\left(x^2\right)\operatorname{cosec}\left(\sqrt{x^2}\right) - 1}{\left(x^2 - 2\right)\operatorname{cosec}\left(\sqrt{x^2}\right)}$$

$$\lim_{x \to \infty} f\left(x^2\right) = \lim_{x \to \infty} \frac{5x^2 \operatorname{cosec} x - 1}{\left(x^2 - 2\right)\operatorname{cosec} x}$$

$$= \lim_{x \to \infty} \frac{x^2 \operatorname{cosec} x \left(5 - \sin x / x^2\right)}{x^2\left(1 - \frac{2}{x^2}\right)\operatorname{cosec} x}$$

$= \lim_{x\to\infty} \frac{\left[5 - \frac{\sin x}{x^2}\right]}{\left(1 - \frac{2}{x^2}\right)}$

Here, $\frac{\sin x}{x^2} \to 0$ as $x \to \infty$

$\Rightarrow \lim_{x\to\infty} \frac{5}{1 - \frac{2}{x^2}} - \lim_{x\to\infty} \frac{\frac{\sin x}{x^2}}{1 - \frac{2}{x^2}}$

$\sin x = \frac{1}{\operatorname{cosec} x}$

$\Rightarrow \lim_{x\to\infty} \frac{5}{1 - 2/x^2} - \lim_{x\to\infty} \frac{\frac{1}{x^2 \operatorname{cosec} x}}{1 - 2/x^2}$

$\Rightarrow 5 - 0 = 5$

Question50

$\lim_{x\to 2} \frac{\sqrt{1+4x} - \sqrt{3+3x}}{x^3 - 8} =$

AP EAPCET 2024 - 21th May Morning Shift

Options:

A. $\frac{1}{72}$

B. $\frac{1}{36}$

C. $\frac{1}{24}$

D. $\frac{1}{12}$

Answer: A

Solution:

We have,

$\lim_{x\to 2} \frac{\sqrt{1+4x} - \sqrt{3+3x}}{x^3 - 8} \qquad \left(\frac{0}{0}\right)$

Using L,Hospital's rule,

$$\lim_{x \to 2} \frac{\frac{1}{2} \frac{4}{\sqrt{1+4x}} - \frac{1}{2} \frac{3}{\sqrt{3+3x}}}{3x^2}$$
$$= \lim_{x \to 2} \frac{4\sqrt{3+3x} - 3\sqrt{1+4x}}{6x^2\sqrt{3+3x}\sqrt{1+4x}}$$
$$= \frac{4 \times 3 - 3 \times 3}{6 \times 4 \times 3 \times 3} = \frac{3}{72 \times 3} = \frac{1}{72}$$

Therefore, the limit is $1/72$.

Question51

If $\lim_{x \to \infty} \frac{(\sqrt{2x+1}+\sqrt{2x-1})^8+(\sqrt{2x+1}-\sqrt{2x-1})^8(Px^4-16)}{\left(x+\sqrt{x^2-2}\right)^8+\left(x-\sqrt{x^2-2}\right)^8} = 1$ **then** $P =$

AP EAPCET 2024 - 21th May Morning Shift

Options:

A. 16

B. 64

C. $\frac{1}{64}$

D. $\frac{1}{16}$

Answer: D

Solution:

$$\lim_{x \to \infty} \frac{\begin{array}{l}[\sqrt{2x+1}+\sqrt{2x-1})^8 \\ +(\sqrt{2x+1}-\sqrt{2x-1})^8\Big] (px^4-18\end{array}}{\left(x+\sqrt{x^2-2}\right)^8+\left(x-\sqrt{x^2-2}\right)^8} = 1$$

Take the ' x ' from numerator and denominator.

$$(x)^4\left(\sqrt{2+\frac{1}{x}}+\sqrt{2-\frac{1}{x}}\right)^8$$

$$\lim_{x \to \infty} \frac{+\left(\sqrt{2+\frac{1}{x}}-\sqrt{2-\frac{1}{x}}\right)^8 \cdot x^4\left(p-\frac{16}{x}\right)}{x^8\left[\left(1+\sqrt{2+\frac{1}{x}}\right)^8+\left(1-\sqrt{2-\frac{1}{x}}\right)\right.} = 1$$

Canceled x^8 from numerator and denominator and put the value of $x \to \infty$

$$\frac{\left\{[\sqrt{(2+0)}+\sqrt{2-0}]^8+[\sqrt{2}-\sqrt{2}]^8\right\}\{P-0}{(1+\sqrt{1})^8+(1-\sqrt{1})^8}=1$$

$$\frac{\left[(\sqrt{2}+\sqrt{2})^8+(0)^8\right]\{P\}}{(2)^8+(0)^8}=1$$

$$\frac{(2\sqrt{2})^8 P}{(2)^8}=1$$

$$P=\frac{(2)^8}{(2\sqrt{2})^8}=\frac{2^8}{2^8\cdot(\sqrt{2})^8}$$

$$P=\frac{1}{2^4}=\frac{1}{16} \Rightarrow P=\frac{1}{16}$$

Question52

$$\lim_{x \to \frac{\pi}{4}} \frac{4\sqrt{2}-(\cos x+\sin x)^5}{1-\sin 2x}=$$

AP EAPCET 2024 - 20th May Evening Shift

Options:

A. $5\sqrt{2}$

B. $3\sqrt{2}$

C. $2\sqrt{2}$

D. $\sqrt{2}$

Answer: A

Solution:

$$\lim_{x \to \frac{\pi}{4}} \frac{4\sqrt{2}-(\cos x+\sin x)^5}{1-\sin 2x}$$

On putting the limit $x \to \frac{\pi}{4}$, we get $\frac{0}{0}$

which is an indeterminate form, using

$$\lim_{x \to \frac{\pi}{4}} \frac{4\sqrt{2} - (\cos x + \sin x)^5}{1 - \sin 2x}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{-5(\cos x + \sin x)^4 \cdot (-\sin x + \cos x)}{-\cos 2x \cdot 2}$$

$$= \frac{5}{2} \lim_{x \to \frac{\pi}{4}} \frac{(\cos x + \sin x)^4 (\cos x - \sin x)}{\cos 2x}$$

$$= \frac{5}{2} \lim_{x \to \frac{\pi}{4}} \frac{(\cos x + \sin x)^4 (\cos x - \sin x)}{\cos^2 x - \sin^2 x}$$

$$= \frac{5}{2} \lim_{x \to \frac{\pi}{4}} (\cos x + \sin x)^3$$

$$= \frac{5}{2} \left(\cos \frac{\pi}{4} + \sin \frac{\pi}{4}\right)^3$$

$$= \frac{5}{2} \times 2\sqrt{2} = 5\sqrt{2}$$

Question53

If $\lim_{x \to 0} \frac{e^x - a - \log(1+x)}{\sin x} = 0$**, then** $a =$

AP EAPCET 2024 - 20th May Evening Shift

Options:

A. 2

B. 0

C. -1

D. 1

Answer: D

Solution:

$$\lim_{x \to 0} \frac{e^x - a - \log(1+x)}{\sin x} = 0$$

$$\text{LHS} = \lim_{x \to 0} \frac{\frac{\{e^x - a - \log(1+x)\}}{x}}{\frac{\sin x}{x}}$$

Dividing by x to both numerator and denominator

$$= \frac{\lim\limits_{x \to 0} \frac{e^x - a}{x} - \frac{\log(1+x)}{x}}{\lim\limits_{x \to 0} \frac{\sin x}{x}}$$

$$= \frac{\lim\limits_{x \to 0} \frac{e^x - a}{x} - \lim\limits_{x \to 0} \frac{\log(1+x)}{x}}{\lim\limits_{x \to 0} \frac{\sin x}{x}}$$

$$= \frac{\lim\limits_{x \to 0} \frac{e^x - a}{x} - 1}{1}$$

$$\left[\because \lim_{x \to 0} \frac{\log(1+x)}{x} = 1, \lim_{x \to 0} \frac{\sin x}{x} = 1\right]$$

$= 0 =$ RHS

$$\Rightarrow \quad \lim_{x \to 0} \frac{e^x - a}{x} - 1 = 0$$

$$\Rightarrow \quad \lim_{x \to 0} \frac{e^x - a}{x} = 1$$

$$\therefore \quad a = 1 \quad \left(\because \lim_{x \to 0} \frac{e^x - 1}{x} = 1\right)$$

Question54

The values of a and b for which the function

$$f(x) = \begin{cases} 1 + |\sin x|^{\frac{a}{\sin x|}} & \frac{-\pi}{6} < x < 0 \\ b, & x = 0 \\ e^{\frac{\tan 2x}{\tan 3x}}, & 0 < x < \frac{\pi}{6} \end{cases} \quad \text{is continuous at } x = 0$$

are

AP EAPCET 2024 - 20th May Evening Shift

Options:

A. $a = 1, b = \frac{2}{3}$

B. $a = \frac{2}{3}, b = e^{\frac{2}{3}}$

C. $a = \frac{2}{3}, b = \frac{3}{2}$

D. $a=-1, b=e^{\frac{2}{3}}$

Answer: B

Solution:

Given,

$$f(x)=\begin{cases}(1+|\sin x|)^{\frac{a}{|\sin x|}}, & -\frac{\pi}{6}<x<0 \\ b, & x=0 \\ e^{\frac{\tan 2x}{\tan 3x}}, & 0<x<\frac{\pi}{6}\end{cases}$$

is continuous at $x=0$

$$\lim_{x\to 0^-} f(x)=f(0)=\lim_{x\to 0^+} f(x)$$

Now,

$$\begin{aligned}\lim_{x\to 0^-}(1+|\sin x|)^{\frac{a}{|\sin x|}} &= e^{\lim_{x\to 0}|\sin x|\cdot\frac{a}{|\sin x|}} \\ &= e^a \\ \lim_{x\to 0^+} e^{\frac{\tan 2x}{\tan 3x}} &= \lim_{x\to 0} e^{\left(\frac{\tan 2x}{2x}\times\frac{3x}{\tan 3x}\times\frac{2x}{3x}\right)} \\ &= \lim_{x\to 0} e^{2/3}=e^{2/3}\end{aligned}$$

Since, f is continuous at $x=0$

$e^a=b=e^{2/3}\Rightarrow a=\frac{2}{3}$

and $b=e^{2/3}$

Question55

If $f(x)=\begin{cases}2x+3, & x\leq 1 \\ ax^2+bx, & x>1\end{cases}$

is differentiable, $\forall x\in R$**, then** $f'(2)=$

AP EAPCET 2024 - 20th May Evening Shift

Options:

A. 5

B. 4

C. -4

D. -10

Answer: C

Solution:

$$f(x)=\begin{cases}2x+3, x\le 1\\ ax^2+bx, x>1\end{cases}$$

Given f is differentiable for all $x \in R$.

$\therefore f$ is continuous everywhere.

At $x=1$

LHL = RHL

$$\lim_{x\to 1^-} f(x)=\lim_{x\to 1^+} f(x)$$

$$\Rightarrow \quad \lim_{x\to 1^-} 2x+3=\lim_{x\to 1^+} ax^2+bx$$

$$\Rightarrow \quad 2(1)+3=a(1)^2+b(1)$$

$$\Rightarrow \quad 5=a+b \quad \ldots \text{(i)}$$

Also, LHD = RHD

$$\lim_{x\to 1^-} f'(x)=\lim_{x\to 1^+} f'(x)$$

$$\Rightarrow \quad 2=\lim_{x\to 1^+} 2ax+b$$

$$\Rightarrow \quad 2=2a+b \quad \ldots \text{(i)}$$

From Eqs. (i) and (ii), we get

$$a=-3, b=8$$

$$\therefore \quad f(x)=\begin{cases}2x+3, & x\le 1\\ -3x^2+8x, & x>1\end{cases}$$

Now, $f'(x)=\begin{cases}2, & x\le 1\\ -6x+8, & x>1\end{cases}$

So, $\quad f'(2)=-6(2)+8=-12+8=-4$

Question56

In the interval $[0,3]$ The function $f(x)=|x-1|+|x-2|$ is

AP EAPCET 2024 - 20th May Evening Shift

Options:

A. discontinuous

B. differentiable

C. continuous but not differentiable at $x=2$ only

D. continuous but not differentiable at $x=1$ and $x=2$

Answer: D

Solution:

Given $f(x)=|x-1|+|x-2|$

$$f(x)=\begin{cases}(1-x)+(2-x), & 0\leq x<1\\(x-1)+(2-x), & 1\leq x<2\\(x-1)+(x-2), & 2\leq x\leq 3\end{cases}$$
$$=\begin{cases}3-2x, & 0\leq x<1\\ \quad 1, & 1\leq x<2\\ 2x-3, & 2\leq x\leq 3\end{cases}$$

At $x=1$

$$\lim_{x\to 1^-}f(x)=\lim_{x\to 1^-}3-2x=3-2(1)=1$$
$$f(1)=1,\ \lim_{x\to 1^+}f(x)=\lim_{x\to 1^+}1=1$$
$$\therefore \lim_{x\to 1^-}f(x)=f(1)=\lim_{x\to 1^+}f(x)$$

Now, $\lim_{x\to 1^-}f'(x)=\lim_{x\to 1^-}(3-2x)'$

$$=\lim_{x\to 1^-}-2=-2$$
$$\lim_{x\to 1^+}f'(x)=0$$
$$\therefore \lim_{x\to 1^-}f'(x)\neq\lim_{x\to 1^+}f'(x)$$

So, $f(x)$ is continuous but not differentiable at $x=1$.

At, $x=2$

$\lim_{x\to 2^-}f(x)=\lim_{x\to 2^-}1=1$

$f(2)=2(2)-3=1$

$$\lim_{x\to 2^+}f(x)=\lim_{x\to 2^+}(2x-3)=4-3=1$$

$\therefore \lim_{x \to 2^-} f(x) = f(2) = \lim_{x \to 2^+} f(x)$

$\lim_{x \to 2^-} f'(x) = 0, \lim_{x \to 2^+} f'(x) = \lim_{x \to 2^+} 2$

$\therefore \lim_{x \to 2^-} f'(x) \neq \lim_{x \to 2^+} f'(x)$

So, $f(x)$ is continuous but not differentiable at $x = 2$

Question57

$\lim_{x \to 0} \frac{\sqrt{1+\sqrt{1+x^4}} - \sqrt{2+x^5+x^6}}{x^4} =$

AP EAPCET 2024 - 20th May Morning Shift

Options:

A. $\frac{1}{4\sqrt{2}}$

B. $\frac{1}{2\sqrt{2}}$

C. $\frac{1}{\sqrt{2}}$

D. $\frac{1}{3\sqrt{2}}$

Answer: A

Solution:

$$\lim_{x \to 0} \frac{\sqrt{1+\sqrt{1+x^4}} - \sqrt{2+x^5+x^6}}{x^4}$$

$$= \lim_{x \to 0} \frac{\left[\sqrt{1+\sqrt{1+x^4}} - \sqrt{2+x^5+x^6}\right]\left[\sqrt{1+\sqrt{1+x^4}} + \sqrt{2+x^5+x^6}\right]}{x^4\left[\sqrt{1+\sqrt{1+x^4}} + \sqrt{2+x^5+x^6}\right]}$$

$$= \lim_{x \to 0} \frac{1+\sqrt{1+x^4} - 2 - x^5 - x^6}{x^4} \times - \lim_{x \to 0} \frac{1}{\sqrt{1+\sqrt{1+x^4}} + \sqrt{2+x^5+x^6}}$$

$$= \lim_{x \to 0} \left[\frac{\sqrt{1+x^4} - 1}{x^4} - \frac{x^5(1+x)}{x^4}\right] \times \frac{1}{2\sqrt{x}}$$

$$= \frac{1}{2\sqrt{2}} \lim_{x \to 0} \frac{\left(\sqrt{1+x^4} - 1\right)\left(\sqrt{1+x^4} + 1\right)}{x^4\left(\sqrt{1+x^4} + 1\right)} = \frac{1}{2\sqrt{2}} \times \frac{1}{2} = \frac{1}{4\sqrt{2}}$$

Question58

$$\lim_{x\to 1}\frac{\sqrt{x}-1}{\left(\cos^{-1}x\right)^2}=$$

AP EAPCET 2024 - 20th May Morning Shift

Options:

A. $-\frac{1}{4}$

B. $\frac{1}{2}$

C. $-\frac{1}{2}$

D. $\frac{1}{4}$

Answer: A

Solution:

$$\lim_{x\to 1}\frac{\sqrt{x}-1}{\left(\cos^{-1}x\right)^2} \qquad \left(\frac{0}{0} form\right)$$

$$=\lim_{x\to 1}\frac{\frac{1}{2\sqrt{x}}}{2\cos^{-1}x\times\left(\frac{-1}{\sqrt{1-x^2}}\right)}=\lim_{x\to 1}\frac{-\sqrt{1-x^2}}{4\sqrt{x}\cos^{-1}x}\left(\frac{0}{0}\right)$$

$$=\frac{-1}{4}\lim_{x\to 1}\frac{\frac{1\times(-2x)}{2\sqrt{1-x^2}}}{\sqrt{x}\times\left(-\frac{1}{\sqrt{1-x^2}}\right)+\cos^{-1}\times\times\frac{1}{2\sqrt{x}}}$$

$$=-\frac{1}{4}\lim_{x\to 1}\frac{\frac{-x}{\sqrt{1-x^2}}}{\frac{-\sqrt{x}}{\sqrt{1-x^2}}+\frac{\cos^{-1}x}{2\sqrt{x}}}=\frac{-1}{4}\lim_{x\to 1}\frac{-x2\sqrt{x}}{-2x+\left(\cos^{-1}x\right)\sqrt{1-x^2}}$$

$$=-\frac{1}{4}$$

Question59

If a function $f(x) = \begin{cases} \frac{\tan(\alpha+1)x+\tan 2x}{x} & \text{if } x > 0 \\ \beta & \text{at } x = 0 \text{ is} \\ \frac{\sin 3x - \tan 3x}{x^3} & \text{if } x < 0 \end{cases}$

continuous at $x = 0$**, then** $|\alpha| + |\beta| =$

AP EAPCET 2024 - 20th May Morning Shift

Options:

A.

60

B.

30

C.

15

D.

45

Answer: B

Solution:

To ensure the function $f(x)$ is continuous at $x = 0$, the left-hand limit (LHL), right-hand limit (RHL), and the value of the function at zero must all be equal:

$\text{LHL} = \text{RHL} = f(0)$

Calculating the Left-Hand Limit (LHL):

For $x < 0$, we have:

$f(x) = \frac{\sin 3x - \tan 3x}{x^3}$

The LHL is evaluated as follows:

$$\lim_{x\to 0^-}\frac{\sin 3x-\tan 3x}{x^3}=\lim_{x\to 0^-}\frac{\sin 3x(\cos 3x-1)}{\cos 3x\times x^3}$$
$$=\lim_{x\to 0^-}\frac{\sin 3x}{x}\times\lim_{x\to 0^-}\frac{1}{\cos 3x}\times\lim_{x\to 0^-}\frac{\cos 3x-1}{x^2}$$
$$=3\times 1\times\lim_{x\to 0^-}(-2)\frac{\sin^2\frac{3x}{2}}{\frac{9x^2}{4}}\times\frac{9}{4}$$
$$=3\times(-2)\times\frac{9}{4}=-\frac{27}{2}$$

Calculating the Right-Hand Limit (RHL):

For $x>0$, the function is:

$f(x)=\frac{\tan((\alpha+1)x)+\tan(2x)}{x}$

The RHL is computed as:

$$\lim_{x\to 0^+}\frac{\tan((\alpha+1)x)+\tan(2x)}{x}=\lim_{x\to 0^+}\frac{\tan((\alpha+1)x)}{x}+\lim_{x\to 0^+}\frac{\tan(2x)}{x}$$
$$=\alpha+1+2=\alpha+3$$

Value at $x=0$:

$f(0)=\beta$

Equating Limits for Continuity:

Given that LHL = RHL = $f(0)$, we have:

$$-\frac{27}{2}=\alpha+3=\beta$$
$$\alpha=-\frac{27}{2}-3=-\frac{33}{2}$$
$$\beta=-\frac{27}{2}$$

Calculate $|\alpha|+|\beta|$:

$|\alpha|=\frac{33}{2},\quad|\beta|=\frac{27}{2}$

$|\alpha|+|\beta|=\frac{33}{2}+\frac{27}{2}=\frac{60}{2}=30$

Question60

$$\lim_{x\to 3}\frac{x^3-27}{x^2-9}=$$

AP EAPCET 2024 - 19th May Evening Shift

Options:

A. $\frac{3}{2}$

B. $\frac{9}{2}$

C. 3

D. 2

Answer: B

Solution:

We have, $\lim_{x \to 3} \frac{x^3 - 27}{x^2 - 9}$

$= \lim_{x \to 3} \frac{(x-3)\left(x^2 + 9 + 3x\right)}{(x-3)(x+3)}$

$= \lim_{x \to 3} \frac{x^2 + 9 + 3x}{x + 3}$

$= \frac{9 + 9 + 9}{6} = \frac{27}{6} = \frac{9}{2}$

Question61

If $f(x) = \begin{cases} 3ax - 2b, & x > 1 \\ ax + b + 1, & x < 1 \end{cases}$ and

$\lim_{x \to 1} f(x)$ exists, then the relation between a and b is

AP EAPCET 2024 - 19th May Evening Shift

Options:

A. $3a - 2b = 1$

B. $2a - 3b = 1$

C. $2a + 3b = 1$

D. $2a + 3b = -1$

Answer: B

Solution:

Given the function $f(x) = \begin{cases} 3ax - 2b, & x > 1 \\ ax + b + 1, & x < 1 \end{cases}$, we need to determine the condition for the existence of the limit $\lim\limits_{x \to 1} f(x)$.

For the limit to exist at $x = 1$, the left-hand limit (LHL) and the right-hand limit (RHL) must be equal:

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$

Calculating the limits from both sides:

Left-hand limit (x approaches 1 from the left):

$LHL = a(1) + b + 1 = a + b + 1$

Right-hand limit (x approaches 1 from the right):

$RHL = 3a(1) - 2b = 3a - 2b$

Setting the LHL equal to the RHL:

$a + b + 1 = 3a - 2b$

Solving for the relationship between a and b:

$a + b + 1 = 3a - 2b$

Rearranging terms yields:

$2a - 3b = 1$

Thus, the relation between a and b when the limit exists is $2a - 3b = 1$.

Question62

The function $f(x) = \begin{cases} \frac{2}{5-x}, & x < 3 \\ 5 - x, & x \geq 3 \end{cases}$ is

AP EAPCET 2024 - 19th May Evening Shift

Options:

A. left discontinuous at $x = 3$

B. right discontinuous at $x = 5$

C. left continuous at $x = 3$

D. discontinuous at $x = 5$

Answer: A

Solution:

We have, $f(x)=\begin{cases}\frac{2}{5-x}, & x<3\\ 5-x, & x\geq 3\end{cases}$

$$\text{LHL}=\lim_{x\to 3^-} f(x)=\lim_{x\to 3^-}\frac{2}{5-x}=1$$
$$\text{RHL}=\lim_{x\to 3^+} f(x)=5-3=2$$

Hence, the function is left discontinuous at $x=3$.

Question63

If $f(x)=\begin{cases}x^{\alpha}\sin\left(\frac{1}{x}\right), & x\neq 0\\ 0, & x=0\end{cases}$

which of the following is true?

AP EAPCET 2024 - 19th May Evening Shift

Options:

A. $f(x)$ is continuous and differentiable if $0\leq\alpha<1$

B. $f(x)$ is discontinuous and not differentiable if $0\leq\alpha<1$

C. $f(x)$ is continuous and differentiable for $\alpha>1$

D. $f(x)$ is discontinuous and differentiable for $\alpha>1$

Answer: C

Solution:

We have, $f(x)=\begin{cases}x^{\alpha}\sin\left(\frac{1}{x}\right), & x\neq 0\\ 0, & x=0\end{cases}$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{x^a \left(\sin \frac{1}{x}\right)}{\left(\frac{1}{x}\right)} \times \frac{1}{x}$$

$$= \lim_{x \to 0} x^{\alpha-1} \left[\because \lim_{x \to 0} \frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}} = 1\right] = 0$$

$\therefore f(x)$ is continuous for all value except -1 .

Now, $f(x)$ is differentiable at $x = 0$, if $\lim_{x \to 0} \frac{f(x)-f(0)}{x-0}$ exists.

$\Rightarrow \quad \lim_{x \to 0} x^{\alpha-1} \sin \frac{1}{x}$ exists finitely.

$\Rightarrow \quad \alpha - 1 > 0$

$\Rightarrow \quad \alpha > 1$

Hence, $f(x)$ is continuous and differentiable for $\alpha > 1$.

Question64

Let $f(x) = \min\left\{x, x^2\right\}$ for every real number of x, then

AP EAPCET 2024 - 19th May Evening Shift

Options:

A. $f(x)$ is continuous for all x

B. $f(x)$ is differentiable for all x

C. $f(x) = 2$ for all $x > 1$

D. $f(x)$ is not differentiable at three values of x

Answer: A

Solution:

For $f(x) = \min\{x, x^2\}$, compare x and x^2:

$$x^2 - x = x(x-1)$$

- If $x < 0$: $x(x-1) > 0 \Rightarrow x^2 > x \Rightarrow \min = x$
- If $0 \le x \le 1$: $x(x-1) \le 0 \Rightarrow x^2 \le x \Rightarrow \min = x^2$
- If $x \ge 1$: $x(x-1) \ge 0 \Rightarrow x^2 \ge x \Rightarrow \min = x$

So

$$f(x) = \begin{cases} x, & x < 0 \\ x^2, & 0 \le x \le 1 \\ x, & x \ge 1 \end{cases}$$

Continuity:

At $x = 0$: both sides give 0.

At $x = 1$: both sides give 1.

Hence f is continuous for all x. ✅

Differentiability:

At $x = 0$: left derivative $= 1$, right derivative $= (2x)_{x=0} = 0 \Rightarrow$ not differentiable.

At $x = 1$: left derivative $= (2x)_{x=1} = 2$, right derivative $= 1 \Rightarrow$ not differentiable.

So f **is not differentiable at exactly two points (0 and 1).**

✅ **Correct option: A.**

Question65

$$\lim_{x \to 0} \frac{1 - \cos x \cdot \cos 2x}{\sin^2 x} =$$

AP EAPCET 2024 - 18th May Morning Shift

Options:

A. $\frac{11}{4}$

B. $\frac{5}{2}$

C. 3

D. 5

Answer: B

Solution:

Step 1: Use series expansion

To simplify the expression, we can use the Taylor series expansions of the trigonometric functions around $x = 0$.

- For $\cos x$, the series is:

$$\cos x = 1 - \frac{x^2}{2} + O(x^4)$$

- For $\cos 2x$, the series is:

$$\cos 2x = 1 - 2x^2 + O(x^4)$$

- For $\sin x$, the series is:

$$\sin x = x - \frac{x^3}{6} + O(x^5)$$

Now, substitute these expansions into the given limit expression.

Step 2: Simplify the numerator

The numerator is $1 - \cos x - \cos 2x$, so:

$$1 - \cos x - \cos 2x = 1 - \left(1 - \frac{x^2}{2} + O(x^4)\right) - \left(1 - 2x^2 + O(x^4)\right)$$

Simplifying:

$$1 - \cos x - \cos 2x = 1 - 1 + \frac{x^2}{2} - 1 + 2x^2 = \frac{5x^2}{2} + O(x^4)$$

Step 3: Simplify the denominator

The denominator is $\sin^2 x$, so:

$$\sin^2 x = \left(x - \frac{x^3}{6} + O(x^5)\right)^2 = x^2 - \frac{x^4}{3} + O(x^6)$$

Step 4: Compute the limit

Now, substitute the simplified expressions into the limit:

$$\lim_{x \to 0} \frac{\frac{5x^2}{2} + O(x^4)}{x^2 - \frac{x^4}{3} + O(x^6)}$$

Factor out x^2 from both the numerator and the denominator:

$$\lim_{x \to 0} \frac{x^2\left(\frac{5}{2} + O(x^2)\right)}{x^2\left(1 - \frac{x^2}{3} + O(x^4)\right)}$$

Cancel x^2 from the numerator and denominator:

$$\lim_{x \to 0} \frac{\frac{5}{2} + O(x^2)}{1 - \frac{x^2}{3} + O(x^4)} = \frac{5}{2}$$

Thus, the correct answer is:

B: $\frac{5}{2}$.

Question66

$$\lim_{x \to \infty}\left(\frac{3x^2 - 2x + 3}{3x^2 + x - 2}\right)^{3x-2} =$$

AP EAPCET 2024 - 18th May Morning Shift

Options:

A. -3

B. e^{-1}

C. e^{-3}

D. -1

Answer: C

Solution:

Step 1: Identify the Form

As $x \to \infty$, the base $\frac{3x^2-2x+3}{3x^2+x-2}$ approaches $\frac{3}{3} = 1$. The exponent $3x - 2$ approaches ∞. This is the indeterminate form 1^∞.

For a limit of the form $\lim_{x\to a}[f(x)]^{g(x)}$, where $f(x) \to 1$ and $g(x) \to \infty$, the solution is:

$$e^{\lim_{x\to a}[f(x)-1]\cdot g(x)}$$

Step 2: Set up the Exponent

Let L be the limit. Then $L = e^k$, where:

$$k = \lim_{x\to\infty}\left(\frac{3x^2-2x+3}{3x^2+x-2} - 1\right)(3x-2)$$

Simplify the expression inside the parentheses:

$$\frac{3x^2-2x+3-(3x^2+x-2)}{3x^2+x-2} = \frac{-3x+5}{3x^2+x-2}$$

Step 3: Solve for k

Now calculate the limit of the product:

$$k = \lim_{x\to\infty}\left(\frac{-3x+5}{3x^2+x-2}\right)(3x-2)$$

$$k = \lim_{x\to\infty}\frac{(-3x+5)(3x-2)}{3x^2+x-2}$$

Expand the numerator:

$$(-3x+5)(3x-2) = -9x^2+6x+15x-10 = -9x^2+21x-10$$

So,

$$k = \lim_{x\to\infty}\frac{-9x^2+21x-10}{3x^2+x-2}$$

Since the degrees of the numerator and denominator are the same (both are x^2), the limit is the ratio of the leading coefficients:

$$k = \frac{-9}{3} = -3$$

Final Answer

Substitute k back into the exponential form:

$$L = e^k = e^{-3}$$

Question67

$$f(x)=\begin{cases}\frac{(2x^2-ax+1)-(ax^2+3bx+2)}{x+1}, & \text{if } x\neq -1\\ k_k, & \text{if } x=-1\end{cases}$$

is a real valued function. If $a, b, k \in R$ and f is continuous on R, then $k =$

AP EAPCET 2024 - 18th May Morning Shift

Options:

A. $-\frac{1}{3}$

B. 6

C. $a-2$

D. $a=3$

Answer: A

Solution:

Step 1: Simplify the function for $x \neq -1$

The function for $x \neq -1$ is:

$$f(x) = \frac{(2x^2 - ax + 1) - (x^2 + 3bx + 2)}{x + 1}$$

Simplify the numerator:

$$2x^2 - ax + 1 - x^2 - 3bx - 2 = x^2 - (a + 3b)x - 1$$

Thus, the function becomes:

$$f(x) = \frac{x^2 - (a + 3b)x - 1}{x + 1}$$

Step 2: Evaluate the limit as $x \to -1$

For $f(x)$ to be continuous at $x = -1$, we need the left-hand limit, right-hand limit, and the function value at $x = -1$ to be the same. First, calculate the limit of $f(x)$ as $x \to -1$:

We need to factor the numerator $x^2 - (a + 3b)x - 1$ so that it cancels with the denominator $x + 1$. This requires that $x + 1$ be a factor of the numerator.

Using synthetic or polynomial division or factoring methods, we can find that:

$$x^2 - (a + 3b)x - 1 = (x + 1)(x - (a + 3b - 1))$$

So, the function becomes:

$$f(x) = \frac{(x + 1)(x - (a + 3b - 1))}{x + 1}$$

For $x \neq -1$, we cancel $x + 1$ from the numerator and denominator, leaving:

$$f(x) = x - (a + 3b - 1)$$

Step 3: Evaluate the limit as $x \to -1$

Now evaluate $f(x)$ as $x \to -1$:

$$\lim_{x \to -1} f(x) = -1 - (a + 3b - 1) = -a - 3b$$

Step 4: Set the limit equal to k

Since $f(x)$ is continuous at $x = -1$, we have:

$$\lim_{x \to -1} f(x) = f(-1) = k$$

Thus, we get the equation:

$$-a - 3b = k$$

Step 5: Determine k

To find k, notice that we have the equation in the options:

$$k = -\frac{1}{3}$$

Thus, the correct answer is **A:** $k = -\frac{1}{3}$.

Question68

If $f(x) = \begin{cases} \frac{2xe^{1/2x} - 3xe^{-1/2x}}{e^{1/2x} + 4e^{-1/2x}} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ is a real valued function, then

AP EAPCET 2024 - 18th May Morning Shift

Options:

A. $f'(0') = \frac{-3}{4}$

B. $f'(0^-) = 2$

C. f is not differentiable at $x = 0$

D. f is differentiable at $x = 0$

Answer: C

Solution:

Given,

$$f(x) = \begin{cases} \frac{2xe^{\frac{1}{2x}} - 3xe^{-\frac{1}{2x}}}{e^{\frac{1}{2x}} + 4e^{-\frac{1}{2x}}} & \text{If } x \neq 0 \\ 0 & \text{If } x = 0 \end{cases}$$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{2xe^{1/2x} - 3xe^{-1/2x}}{e^{1/2x} + 4e^{-1/2x}}$$

$$= \lim_{x \to 0} \frac{x\left(2e^{1/x} - 3\right)}{e^{1/x} + 4} = 0$$

$\therefore f(x)$ is continuous of $x=0$

$$f'\left(0^{-}\right)=\lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}$$

LHD

$$=\lim_{h\to 0}\frac{h\left(-2e^{-1/h}-3\right)}{-h\left(e^{-1/h}+4\right)}$$

$$=\lim_{h\to 0}\frac{2e^{-1/h}-3}{e^{-1/h}+4}=\frac{-3}{4}$$

$$f\left(0^{-}\right)=\frac{-3}{4}$$

$$\text{RHD }=f'\left(0^{+}\right)=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$$

$$=\lim_{h\to 0}\frac{h\left(2e^{1/h}-3\right)}{h\left(e^{1/h}+4\right)}$$

LHD $\neq$ RHD $\quad\therefore f$ is not different of $x=0$

$$=\lim_{h\to 0}\frac{e^{1/h}\left(2-\frac{3}{e^{1/h}}\right)}{e^{1/h}\left(1+\frac{4}{e^{1/h}}\right)}=2$$

$$\text{RHD }=f'\left(0^{-}\right)=2$$

Question69

$$\lim_{x\to -\infty}\log_e(\cosh x)+x=$$

AP EAPCET 2022 - 5th July Morning Shift

Options:

A. $\log 2$

B. $-\log 2$

C. $\log\left(\frac{1}{2}\right)+2$

D. $\log\left(\frac{1}{2}\right)-2$

Answer: B

Solution:

Here, $\lim_{x \to -\infty} \log_e(\cosh x) + x$

$= \lim_{x \to -\infty} \log\left(e^x + e^{-x}\right) + \ln 2$

$\Rightarrow \quad \lim_{x \to -\infty} \log\left(1 + e^{2x}\right) + \ln 2$

$= -\ln 2$

Question70

If a, b and c are three distinct real numbers and $\lim_{x \to \infty} \frac{(b-c)x^2+(c-a)x+(a-b)}{(a-b)x^2+(b-c)x+(c-a)} = \frac{1}{2}$, then $a + 2c =$

AP EAPCET 2022 - 5th July Morning Shift

Options:

A. b

B. 2b

C. 3b

D. 4b

Answer: C

Solution:

$$\lim_{x \to \infty} \frac{(b-c)x^2 + (c-a)x + (a-b)}{(a-b)x^2 + (b-c)x + (c-a)} = \frac{1}{a}$$

$$= \lim_{x \to \infty} \frac{x^2}{x^2} \left[\frac{(b-c) + (c-a)\frac{1}{x} + \frac{(a-b)}{x^2}}{(a-b) + (b-c)\frac{1}{x} + \frac{(c-a)}{x^2}} \right] = \frac{1}{2}$$

$$= \frac{b-c}{a-b} = \frac{1}{2}$$

$\Rightarrow 2b - 2c = a - b$

$\Rightarrow 3b = a + 2c$

Question71

$$\lim_{x \to -\infty} \frac{3|x|-x}{|x|-2x} - \lim_{x \to 0} \frac{\log(1+x^3)}{\sin^3 x} =$$

AP EAPCET 2022 - 5th July Morning Shift

Options:

A. $\frac{1}{3}$

B. $-\frac{1}{4}$

C. 2

D. $-\frac{5}{3}$

Answer: A

Solution:

$$\lim_{x \to -\infty} \frac{3|x| - x}{|x| - 2x} - \lim_{x \to 0} \frac{\log\left(1 + x^3\right)}{\sin^3 x}$$

$$\lim_{x \to -\infty} \frac{-3x - x}{-x - 2x} - \lim_{x \to 0} \frac{\log\left(1 + x^3\right)}{x^3 \frac{(\sin x^3 x)}{x^3}}$$

$$\Rightarrow \lim_{x \to -\infty} \frac{-4x}{-3x} - \lim_{x \to 0} \frac{1}{\left(\frac{\sin^3 x}{x^3}\right)}$$

$$\lim_{x \to -\infty} \frac{4}{3} - 1 = \frac{1}{3}$$

Question72

If $[\cdot]$ denotes greatest integer function, then $\lim_{x \to \frac{-3}{5}} \frac{1}{\dot{x}} \left[\frac{-1}{x}\right] =$

AP EAPCET 2022 - 4th July Evening Shift

Options:

A. $-5/3$

B. $5/3$

C. $10/3$

D. $-10/3$

Answer: A

Solution:

$$\lim_{x \to \frac{-3}{5}} \frac{1}{x}\left[-\frac{1}{x}\right]$$
$$= \frac{1}{\frac{-3}{5}}\left[-\frac{1}{\left(-\frac{3}{5}\right)}\right] = \frac{-5}{3}[1.66]$$
$$= \frac{-5}{3} \cdot (1) = \frac{-5}{3}$$

Question73

If $l, m(l < m)$ are roots of $ax^2 + bx + c = 0$, then $\lim_{x \to \alpha} \frac{|ax^2+bx+c|}{ax^2+bx+c} =$

AP EAPCET 2022 - 4th July Evening Shift

Options:

A. $\frac{|a|}{a}, \forall \alpha \in R$

B. $\frac{-|a|}{a}$, when $\alpha \notin (l, m)$

C. $\frac{-|a|}{a}$, when $\alpha \in (1, m)$

D. $\frac{|a|}{a}$, when $\alpha \in (i, m)$

Answer: D

Solution:

If l and m (where $l < m$) are the roots of the quadratic equation $ax^2 + bx + c = 0$, then the following limit expression is considered:

$\lim_{x \to \alpha} \frac{|ax^2+bx+c|}{ax^2+bx+c}$

Given that $\alpha \in (l, m)$, by applying L'Hôpital's rule, we analyze the limit:

$\lim_{x \to \alpha} \frac{|ax^2+bx+c|}{ax^2+bx+c}$

This results in:

$\frac{|a|}{a}$, when $\alpha \in (l, m)$

Hence, the above contributes to the solution for the considered limit.

Question74

Let $f(x) = \begin{cases} \frac{1}{|x|}, & \text{for } |x| > 1 \\ ax^2 + b, & \text{for } |x| \leq 1 \end{cases}$. If $\lim_{x \to 1^+} f(x)$ and $\lim_{x \to 1^-} f(x)$ exist, then the possible values for a and b are

AP EAPCET 2022 - 4th July Evening Shift

Options:

A. $a = b = 1$

B. $a = -1/2, b = -3/2$

C. $a = 3/2, b = -1/2$

D. $a = 1/2, b = -3/2$

Answer: C

Solution:

$$\text{Let } f(x) = \begin{cases} \frac{1}{|x|}, & |x| > 1 \\ ax^2 + b, & |x| \leq 1 \end{cases}$$
$$= \begin{Bmatrix} \frac{1}{x}, & x > 1 \\ ax^2 + b & x \leq 1 \end{Bmatrix}$$

Given, that $\lim_{x \to 1^+} f(x)$ and $\lim_{x \to 1^+} f(x)$ exist.

Thus, $\lim\limits_{x \to 1^+} (x) = \lim\limits_{x \to 1^+} \frac{1}{x} = 1$

$\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-} \left(ax^2 + b\right) = a + b$

According to question, $a + b = 1$ if take $a = \frac{3}{2}$ and $b = -\frac{1}{2}$.

Then, option (c) is only true all other option are incorrect.

Question75

$\frac{d}{dx}\left(\lim_{x \to 2} \frac{1}{y-2}\left(\frac{1}{x} - \frac{1}{x+y-2}\right)\right) =$

AP EAPCET 2022 - 4th July Evening Shift

Options:

A. $1/x^2$

B. $2/x^3$

C. $-2/x^3$

D. $1/x^3$

Answer: C

Solution:

$$\frac{d}{dx}\left(\lim_{y \to 2} \frac{1}{y-2}\left(\frac{1}{x} - \frac{1}{x+y-2}\right)\right)$$
$$= \frac{d}{dx} \lim_{y \to 2}\left(\frac{\left(\frac{1}{x} - \frac{1}{x+y-2}\right)}{(y-2)}\right)$$

Applying L Hospital's rule,

$$\frac{d}{dx} \lim_{y \to 2} \frac{0 + \frac{1}{(x+y-2)^2}}{1}$$
$$= \frac{d}{dx}\left(\frac{1}{x^2}\right) = -\frac{2}{x^3}$$

Question76

If $f(x)=\begin{cases}\frac{x^2\log(\cos x)}{\log(1+x)} & , \quad x\neq 0\\ 0 & , x=0\end{cases}$, then at $x=0, f(x)$ is

AP EAPCET 2022 - 4th July Evening Shift

Options:

A. not continuous

B. continuous but not differentiable

C. differentiable

D. not continuous, but differentiable

Answer: C

Solution:

Given, $\begin{cases}\frac{x^2\log(\cos x)}{\log(1+x)} & , x\neq 0\\ 0 & , x=0\end{cases}$

Now, $\lim\limits_{x\to 0}\frac{f(x)-f(0)}{x-0}$

$$=\lim_{x\to 0}\frac{x^2\log(\cos x)}{x\log(1+x)}=\lim_{x\to 0}\frac{x\cdot\log(\cos x)}{\log(1+x)}$$
$$=\lim_{x\to 0}\frac{x\log[1-(1-\cos x)]}{1-\cos x}\cdot\frac{1-\cos x}{\log(1+x)}$$
$$=\lim_{x\to 0}\frac{\log[1-(1-\cos x)]}{1-\cos x}\cdot\frac{2\sin^2\left(\frac{x}{2}\right)}{4\left(\frac{x}{2}\right)^2}, x^2\frac{x}{\log(1+x)}=0$$

So, f is differentiable hence it is also continuous.

Question77

Let $f:R^+\longrightarrow R^+$ be a function satisfying $f(x)-x=\lambda$ (constant), $\forall x\in R^+$ and $f(xf(y))=f(xy)+x, \forall x,y,\in R^+$. Then,

$$\lim_{x \to 0} \frac{(f(x))^{1/3}-1}{(f(x))^{1/2}-1} =$$

AP EAPCET 2022 - 4th July Morning Shift

Options:

A. 1/3

B. 0

C. 2/3

D. 1

Answer: C

Solution:

$f(x \cdot f(y)) = f(xy) + x$ ... (i)

$x \leftrightarrow y$

$\Rightarrow \quad f(y \cdot f(x)) = f(xy) + y$ ... (ii)

$x \leftrightarrow f(x)$ in Eq. (i),

$f(f(x) \cdot f(y)) = f(y \cdot f(x)) + f(x)$ ... (iii)

$\Rightarrow \quad f(fx) \cdot f(y)) = f(xy) + y + f(x)$

[from Eq. (ii)] .... (iv)

Again $x \leftrightarrow y$ in Eq. (iv),

$f(f(y) \cdot f(x)) = f(yx) + x + f(y)$ ... (v)

Eq. (iv) = Eq. (v),

$f(xy) + y + f(x) = f(xy) + x + f(y)$

$\Rightarrow f(x) - x = f(y) - y$ .... (vi)

$\Rightarrow f(x) - x = \lambda = f(y) - y$

Substituting $f(x) = \lambda + x$ in Eq. (i), we have

$x \cdot f(y) + \lambda = (xy + \lambda) + x$

$\Rightarrow \quad xf(y) = xy + x \quad [\because f(y) = \lambda + y]$

$\therefore x(y + \lambda) = xy + x$

$\Rightarrow \lambda x = x$

$\Rightarrow \lambda = 1 \quad [\because x > 0]$

$\therefore \quad f(x) = x + \lambda = x + 1$

Now, $\lim_{x \to 0} \frac{(f(x))^{\frac{1}{3}}-1}{(f(x))^{\frac{1}{2}}-1} = \lim_{x \to 0} \frac{(1+x)^{\frac{1}{3}}-1}{(1+x)^{\frac{1}{2}}-1}$

$$= \lim_{x \to 0} \left(\frac{(1+x)^{\frac{1}{3}}-1}{1+x-1} \right) \left(\frac{1+x-1}{(1+x)^{\frac{1}{2}}-1} \right)$$

$$= \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

Question78

$$\text{If } \lim_{x \to 0} \frac{|x|}{\sqrt{x^4+4x^2+5}} = k$$

$$\lim_{x \to 0} x^4 \sin\left(\frac{1}{3\sqrt{x}}\right) = l. \text{ Then, } k+l =$$

AP EAPCET 2022 - 4th July Morning Shift

Options:

A. 0

B. 1

C. -1

D. 5

Answer: A

Solution:

$$k = \lim_{x \to 0} \frac{|x|}{\sqrt{x^4+4x^2+5}}$$

$$= \frac{|0|}{\sqrt{5}} = 0$$

$$l = \lim_{x \to 0} x^4 \sin\left(\frac{1}{3\sqrt{x}}\right) = (0)^4 \sin\left(\frac{1}{3\sqrt{0}}\right)$$

$$= 0 \times (\text{ a value which belongs to } [-1, 1])$$

$$= 0$$

$$\therefore k + l = 0 + 0 = 0$$

Question79

If $\lim_{n \to \infty} x^n \log_e x = 0$**, then** $\log_x 12 =$

AP EAPCET 2022 - 4th July Morning Shift

Options:

A. negative

B. positive

C. zero

D. any value between -1 and 1

Answer: A

Solution:

$\lim_{n \to \infty} x^n \log_e x = 0$

It is possible only when $x \in (0, 1)$ i.e. $x > 0$ and $x < 1$.

In this case,

$$x^n \text{ or } x^\infty = 0$$
$$\Rightarrow \quad x^\infty \log_c x = 0$$

Now, $\log_x 12$:

We know, for $\log_a b$: If $a \in (0, 1)$, then $\log_a b =$ Negative

$\therefore \log_x 12 =$ Negative \quad $[\because x \in (0, 1)]$

Question80

If $f(x) = \text{Max}\{3 - x, 3 + x, 6\}$ **is not differentiable at** $x = a$**, and** $x = b$**, then** $|a| + |b| =$

Options:

A. 4

B. 5

C. 6

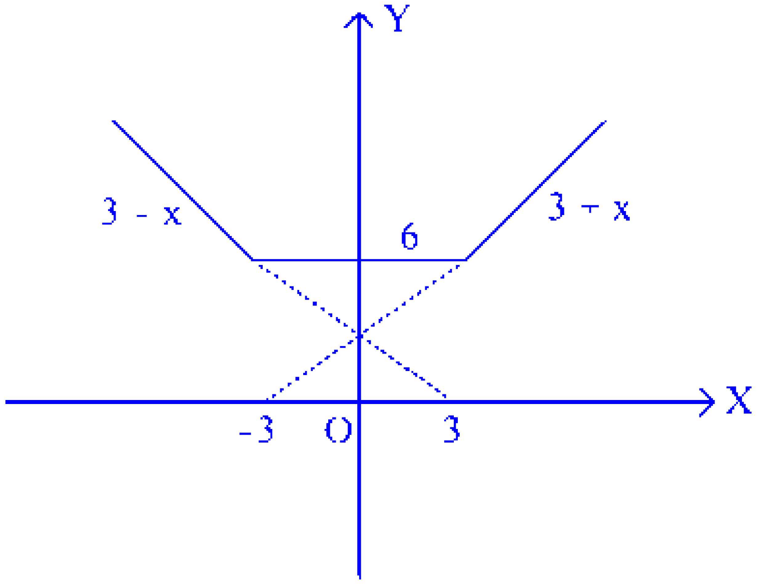
D. 8

Answer: C

Solution:

$f(x) = \text{Max}\{3 - x, 3 + x, 6\}$

Plotting the graph :



We know that a function is not differentiable at the points where it has sharp corner point.

In the above graph, we can see that only two sharp corner points are present.

Therefore, the above function is not differentiable at two points which are

$$x = -3 \text{ and } x = 3$$
$$a = -3 \text{ and } b = 3$$
$$\therefore |a| + |b| = 3 + 3 = 6$$

Question81

$\lim\limits_{n \to \infty}\left(\frac{1}{1^5+n^5}+\frac{2^4}{2^5+n^5}+\frac{3^4}{3^5+n^5}+\ldots+\frac{n^4}{n^5+n^5}\right)=$

AP EAPCET 2022 - 4th July Morning Shift

Options:

A. $\frac{1}{5}\log 3$

B. $\frac{1}{3}\log 5$

C. $\frac{1}{2}\log 5$

D. $\log \sqrt[5]{2}$

Answer: D

Solution:

$$\begin{aligned}
&\lim_{n \to \infty}\left(\frac{1^4}{1^5+n^5}+\frac{2^4}{2^5+n^5}+\frac{3^4}{3^5+n^5}+\ldots\right.\\
&= \quad \left.+\frac{n^4}{n^5+n^5}\right)\\
&= \lim_{n \to \infty}\sum_{r=1}^{n}\frac{r^4}{r^5+n^5}\\
&= \lim_{n \to \infty}\sum_{r=1}^{n}\frac{(r/n)^4}{(r/n)^5+1}\cdot\left(\frac{1}{n}\right)
\end{aligned}$$

On putting $\frac{r}{n} \to x, \frac{1}{n} \to dx$ and $\Sigma \to \int$

$$\begin{aligned}
&= \int_0^1 \frac{x^4}{1+x^5}\cdot dx = \frac{1}{5}\int_0^1 \frac{5x^4}{1+x^5}\cdot dx\\
&= \frac{1}{5}\left[\log\left(1+x^5\right)\right]_0^1 = \frac{1}{5}[\log 2 - \log 1]\\
&= \frac{1}{5}[\log 2 - 0] = \frac{1}{5}\log 2\\
&= \log \sqrt[5]{2}
\end{aligned}$$

Question82

If $\lim_{x \to 0}\left(\frac{11x^3-3x+4}{13x^3-5x^2-7}\right)=\frac{a}{b}$, then the value of $a+b$ equals

AP EAPCET 2021 - 20th August Evening Shift

Options:

A. 11

B. 13

C. 8

D. 24

Answer: D

Solution:

$$\lim_{x \to 0}\left(\frac{11x^3-3x+4}{13x^3-5x^2-7}\right)$$
$$=\frac{11(0)-3(0)+4}{13(0)-5(0)-7}=\frac{+4}{-7}=\frac{a}{b}$$
$$\Rightarrow \quad a=4\alpha \text{ and } b=-7\alpha$$
$$\Rightarrow a+b=-3\alpha \text{ or } a+b=3\alpha$$

Then, any multiple of 3 will be equal to $a+b$.

Among options 24 is multiple of 3.

$\therefore$ 24 is required value of $a+b$.

Question83

$$\lim_{x \to 1}\frac{(1-x)(1-x^2)\ldots(1-x^{2n})}{\{(1-x)(1-x^2)\ldots\ldots(1-x^n)\}^2}=\_\_\_\_\_\_\_\_\_\_\_\_\_\_, \forall n \in N$$

AP EAPCET 2021 - 20th August Evening Shift

Options:

A. ${}^{2n}P_n$

B. ${}^{2n}\mathrm{C}$

C. $(2n)!$

D. $\frac{(2n)!}{n!}$

Answer: B

Solution:

$$\begin{aligned}
&\lim_{x \rightarrow 1} \frac{(1-x)\left(1-x^2\right) \ldots\left(1-x^{2n}\right)}{\left\{(1-x)\left(1-x^2\right) \ldots\left(1-x^n\right)\right\}^2} \\
&=\lim_{x \rightarrow 1} \frac{(x-1)\left(x^2-1\right) \ldots\left(x^{2n}-1\right)}{\left\{(x-1)\left(x^2-1\right) \ldots\left(x^n-1\right)\right\}^2} \\
&=\lim_{x \rightarrow 0} \frac{(x-1)\left(x^2-1\right) \ldots\left(x^{2n}-1\right)}{\left\{\frac{(x-1)}{(x-1)} \cdot \frac{\left(x^2-1\right)}{(x-1)} \ldots \frac{\left(x^n-1\right)}{(x-1)}\right\}^2 \cdot(x-1)^{2n}} \\
&=\lim_{x \rightarrow 1} \frac{\frac{(x-1)}{(x-1)} \frac{\left(x^2-1\right)}{(x-1)} \ldots \frac{\left(x^{2n}-1\right)}{(x-1)}}{\left\{\frac{(x-1)}{(x-1)} \cdot \frac{\left(x^2-1\right)}{(x-1)} \ldots \frac{\left(x^n-1\right)}{(x-1)}\right\}^2} \\
&=\frac{1 \cdot 2 \cdot 3 \cdot \ldots 2n}{\{1 \cdot 2 \ldots n\}^2} \\
&=\frac{(2n)!}{(n!)^2}=\frac{(2n)!}{n!n!}={ }^{2n} C_n\left[\because \lim_{x \rightarrow a} \frac{x^n-a^n}{x-a} n a^{n-1}\right]
\end{aligned}$$

Question84

If $f(x)=\frac{\log_e\left(1+x^2(\tan x)\right)}{\sin x^3}, x \neq 0$ is to be continuous at $x=0$, then $f(0)$ must be equal to

AP EAPCET 2021 - 20th August Evening Shift

Options:

A. 1

B. 0

C. $\frac{1}{2}$

D. -1

Answer: A

Solution:

$f(x)=\frac{\log_e(1+x^2\tan x)}{\sin x^3}\cdot x\neq 0$

f is continuous at $x=0$, then

$f(0)=\text{LHL}=\text{RHL}$

$\text{RHL}=\lim\limits_{h\to 0}\frac{\log_e(1+h^2\tan h)}{\sin h^3}$

$$
\begin{aligned}
&=\lim_{h\to 0}\frac{\log_e(1+h^2\tan h)}{h^3\cdot\left(\frac{\sin h^3}{h^3}\right)}\\
&=\lim_{h\to 0}\frac{\log_e(1+h^2\tan h)}{h^3} \qquad \left[\because \lim_{h\to 0}\frac{\sin h}{h}=1\right]\\
&=\lim_{h\to 0}\frac{\log_e\left(1+h^3\frac{\tan h}{h}\right)}{h^3}\\
&=\lim_{h\to 0}\frac{\log_e(1+h^3)}{h^3} \qquad \left(\because \lim_{h\to 0}\frac{\tan h}{h}=1\right)\\
&=\lim_{h\to 0}\frac{\frac{1}{(1+h^3)}(3h^2)}{3h^2}\\
&=\lim_{h\to 0}\frac{1}{1+h^3}=1
\end{aligned}
$$

$\therefore f(0)\ =1$

Question85

$\lim\limits_{n\to\infty}\frac{n(2n+1)^2}{(n+2)(n^2+3n-1)}$ **is equal to**

AP EAPCET 2021 - 20th August Morning Shift

Options:

A. 0

B. 4

C. 2

D. ∞

Answer: B

Solution:

$$\lim_{n\to\infty} \frac{n(2n+1)^2}{(n+2)(n^2+3n-1)}$$

$$= \frac{\text{Coefficient of } n^3 \text{ in numerator}}{\text{Coefficient of } n^3 \text{ in denominator}} = \frac{4}{1}$$

Question86

If the function $f(x)$, defined below, is continuous on the interval $[0,8]$, then $f(x) = \begin{cases} x^2+ax+b & , \quad 0 \le x < 2 \\ 3x+2, & \quad 2 \le x \le 4 \\ 2ax+5b & , 4 < x \le 8 \end{cases}$

AP EAPCET 2021 - 20th August Morning Shift

Options:

A. $a = 3, b = -2$

B. $a = -3, b = 2$

C. $a = -3, b = -2$

D. $a = 3, b = 2$

Answer: A

Solution:

$\lim_{x\to 2^-} f(x) = f(2)$

$\because f(x)$ is continuous on $[0,8]$.

$\Rightarrow f(x)$ is continuous at 2 and 4 as well

$$4 + 2a + b = 8$$
$$\Rightarrow 2a + b = 4 \quad \ldots\ldots (i)$$
$$\lim_{x \to 4^+} f(x) = f(4)$$
$$8a + 5b = 14 \quad \ldots\ldots (ii)$$

From Eqs. (i) and (ii), we get

$a = 3, b = -2$

Question87

If $f(x)$, defined below, is continuous at $x = 4$, then

$$f(x) = \begin{cases} \frac{x-4}{|x-4|} + a & , \quad x < 4 \\ a + b & , \quad x = 4 \\ \frac{x-4}{|x-4|} + b & , \quad x > 4 \end{cases}$$

AP EAPCET 2021 - 20th August Morning Shift

Options:

A. $a = 0$ and $b = 0$

B. $a = 1$ and $b = 1$

C. $a = -1$ and $b = 1$

D. $a = 1$ and $b = -1$

Answer: D

Solution:

$$\lim_{x \to 4^-} f(x) = f(4) = \lim_{x \to 4^-} f(x)$$

$$-1 + a = a + b = 1 + b$$

$$\Rightarrow b = -1, a = 1$$

Question88

If $f(x)=\begin{cases}\frac{e^{\alpha x}-e^{x}-x}{x^{2}}, & x \neq 0 \\ \frac{3}{2}, & x=0\end{cases}$

Find the value of α for which the function f is continuous

AP EAPCET 2021 - 19th August Evening Shift

Options:

A. 1

B. 0

C. 4

D. 2

Answer: D

Solution:

f is continuous $\Rightarrow f(x)$ will be continuous at $x=0$

$$\begin{aligned} & \Rightarrow \lim_{x \rightarrow 0} f(x)=f(0) \\ & \Rightarrow \lim_{x \rightarrow 0} \frac{e^{\alpha x}-e^{x}-x}{x^{2}}=\frac{3}{2} \\ & \Rightarrow \lim_{x \rightarrow 0} \frac{\alpha e^{\alpha x}-e^{x}-1}{2 x}=\frac{3}{2}\end{aligned}$$

$$

[By L hospital's rule]

This is solvable when

$\alpha e^{0}-e^{0}-1=0 \Rightarrow \alpha-2=0$

or $\alpha=2$

Question89

The value of $k(k>0)$, for which the function $f(x)=\frac{(e^x-1)^4}{\sin\left(\frac{x^2}{k^2}\right)\log\left(1+\frac{x^2}{2}\right)}$, where $x\neq 0$ and $f(0)=8$

AP EAPCET 2021 - 19th August Evening Shift

Options:

A. 1

B. 4

C. 2

D. 3

Answer: C

Solution:

$$f(0)=\lim_{x\to 0}\frac{(e^x-1)^4}{\sin\left(\frac{x^2}{k^2}\right)\log\left\{1+\frac{x^2}{2}\right\}}=8$$

$$\Rightarrow \lim_{x\to 0}\frac{2k^2\left(\frac{e^x-1}{x}\right)^4}{\frac{\sin\left(\frac{x^2}{k^2}\right)}{\left(\frac{x^2}{k^2}\right)}\cdot\frac{\log\left\{1+\frac{x^2}{2}\right\}}{\frac{x^2}{2}}}=8$$

$$\left\{\begin{array}{l}\because \lim_{x\to}\frac{e^x-1}{x}=1\Rightarrow\lim_{x\to 0}\frac{\sin x}{x}=1\\ \lim_{x\to 0}\frac{\log(1+x)}{x}=1\end{array}\right\}$$

$$\Rightarrow\quad 2k^2=8$$

$$\Rightarrow\quad k=\pm 2$$

Question90

If $f''(x)$ is continuous at $x=0$ and $f''(0)=4$, then find the following value.$\lim_{x\to 0}\frac{2f(x)-3f(2x)+f(4x)}{x^2}$ is equal to

AP EAPCET 2021 - 19th August Evening Shift

Options:

A. 4

B. 8

C. 12

D. 16

Answer: C

Solution:

$$\begin{aligned}
&\lim_{x \to 0} \frac{2f(x) - 3f(2x) + f(4x)}{x^2} \quad [0/0 \text{ form}] \\
&= \lim_{x \to 0} \frac{2f'(x) - 6f'(2x) + 4f'(4x)}{2x} \quad [0/0 \text{ form}] \\
&= \lim_{x \to 0} \frac{2f''(x) - 12f''(2x) + 16f''(4x)}{2} \\
&= \frac{2f''(0) - 12f''(0) + 16f''(0)}{2} \\
&= 3f''(0) = 3 \cdot 4 = 12 \\
&[\because f''(0) = 4]
\end{aligned}$$

Question91

$\lim_{z \to 1} \frac{z^{(1/3)} - 1}{z^{(1/6)} - 1}$ **is equal to**

AP EAPCET 2021 - 19th August Morning Shift

Options:

A. -1

B. 1

C. 2

D. -2

Answer: C

Solution:

$\lim_{z \to 1} \frac{z^{1/3}-1}{z^{1/6}-1} \left[\frac{0}{0} \text{ form}\right]$

$= \lim_{z \to 1} \frac{\frac{1}{3} z^{-2/3}}{\frac{1}{6} z^{-5/6}}$ [L' Hospital rule]

$= \frac{6}{3} \frac{(1)^{5/6}}{(1)^{2/3}} = \frac{6}{3} = 2$

Question92

$$f(x) = \begin{cases} \frac{72^x - 9^x - 8^x + 1}{\sqrt{2} - \sqrt{1+\cos x}}, & x \neq 0 \\ K \log 2 \log 3, & x = 0 \end{cases}$$

Find the value of k for which the function f is continuous.

AP EAPCET 2021 - 19th August Morning Shift

Options:

A. $\sqrt{2}$

B. 24

C. $18\sqrt{3}$

D. $24\sqrt{2}$

Answer: D

Solution:

f is continuous at $x = 0$

$$\therefore \lim_{x \to 0} f(x) = f(0)$$

$$\Rightarrow \lim_{x \to 0} \frac{72^x - 9^x - 8^x + 1}{\sqrt{2} - \sqrt{1 + \cos x}} = k \log 2 \log 3$$

$$\Rightarrow k \log 2 \cdot \log 3$$

$$= \lim_{x \to 0} \frac{(9^x - 1)(8^x - 1)(\sqrt{2} + \sqrt{1 + \cos x})}{2 - (1 + \cos x)}$$

$$= \lim_{x \to 0} \left[\begin{array}{c} \frac{(9^x - 1)}{x} \cdot \left(\frac{8^x - 1}{x}\right) \cdot \frac{x^2}{1 - \cos x} \\ \cdot(\sqrt{2} + \sqrt{1 + \cos x}) \end{array} \right]$$

$$= \log 9 \cdot \log 8 \cdot 2(\sqrt{2} + \sqrt{2})$$

$$= 2 \log 3 \cdot 3 \log 2 \cdot 4\sqrt{2}$$

$$\Rightarrow k \log 3 \cdot \log 2 = 24\sqrt{2} \log 3 \cdot \log 2 \Rightarrow k = 24\sqrt{2}$$

Question93

If the function $f(x)$, defined below is continuous in the interval $[0, \pi]$

, then $f(x) = \begin{cases} x + a\sqrt{2}(\sin x) & , \quad 0 \leq x < \frac{\pi}{4} \\ 2x(\cot x) + b, & \frac{\pi}{4} \leq x \leq \frac{\pi}{2} \\ a(\cos 2x) - b(\sin x), & \frac{\pi}{2} < x \leq \pi \end{cases}$

AP EAPCET 2021 - 19th August Morning Shift

Options:

A. $a = \frac{\pi}{6}, b = \frac{\pi}{12}$

B. $a = \frac{-\pi}{6}, b = \frac{\pi}{12}$

C. $a = \frac{-\pi}{6}, b = \frac{-\pi}{12}$

D. $a = \frac{\pi}{6}, b = \frac{-\pi}{12}$

Answer: D

Solution:

$\because f$ is continuous in $(0, \pi)$. $\Rightarrow f$ is continuous at $x = \frac{\pi}{4}, \frac{\pi}{2}$

$\therefore$ LHL of $f(x)$ (at $x=\frac{\pi}{4}$) $=$ RHL of $f(x)$ (at $x=\frac{\pi}{4}$) $=f\left(\frac{\pi}{4}\right)$

$$\Rightarrow \quad \lim_{x \to \frac{\pi}{4}^-} f(x)=\lim_{x \to \frac{\pi}{4}^+} f(x)=2\left(\frac{\pi}{4}\right) \cot \frac{\pi}{4}+b$$

$$\Rightarrow \quad \lim_{x \to \frac{\pi}{4}^-}(x+a \sqrt{2} \sin x)=\lim_{x \to \frac{\pi}{4}^+}(2 x \cot x+b)$$

$$=\frac{\pi}{2}+b$$

$$\Rightarrow \frac{\pi}{4}+a \sqrt{2} \sin \frac{\pi}{4}=2 \cdot \frac{\pi}{4} \cot \frac{\pi}{4}+b$$

$\Rightarrow a-b=\frac{\pi}{4}$ ..... (i)

Again, LHL of $f(x)$ (at $x=\frac{\pi}{2}$) $=$ RHL of $f(x)$ (at $x=\frac{\pi}{2}$) $\quad \{\because f(x)$ is continuous at $x=\frac{\pi}{2}\}$

$$\Rightarrow \quad \lim_{x \to \frac{\pi}{2}^-} f(x)=\lim_{x \to \frac{\pi}{2}^+} f(x)$$

$$\Rightarrow \quad \lim_{x \to \frac{\pi}{2}^-}(2 x \cot x+b)$$

$$=\lim_{x \to \frac{\pi}{2}^+}(a \cos 2 x-b \sin x)$$

$$\Rightarrow \quad 2 \cdot \frac{\pi}{2} \cot \frac{\pi}{2}+b=a \cos \pi-b \sin \frac{\pi}{2}$$

$\Rightarrow a=-2 b$ ..... (ii)

On solving Eqs. (i) and (ii), we get

$a=\frac{\pi}{6}, b=-\frac{\pi}{12}$
